

— Integration WKst

1) $\int \frac{\ln x}{x} dx$ $u = \ln x$
 $du = \frac{1}{x} dx$

$$\int u du = \frac{u^2}{2} + C$$

$$\frac{(\ln x)^2}{2} + C$$

4) $\int x^2 \sec^2(x^3) dx$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\frac{1}{3} \int \sec^2 u du$$

$$= \frac{1}{3} \tan u + C$$

$$\frac{1}{3} \tan(x^3) + C$$

2) $\int \frac{2x}{5x^2+7} dx$

$$u = 5x^2 + 7$$

$$du = 10x dx$$

$$\frac{1}{5} du = 2x dx$$

$$\frac{1}{5} \int \frac{1}{u} du = \frac{\ln|u|}{5} + C$$

$$\frac{\ln|5x^2+7|}{5} + C$$

5) $\int \sin^2 x dx$

$$= \frac{1}{2} \int 1 - \cos(2x) dx$$

$$\frac{1}{2} \left[\int 1 dx - \int \cos(2x) dx \right]$$

3) $\int \frac{e^{1/x}}{x^2} dx$

$$u = \frac{1}{x} = x^{-1}$$

$$du = -1x^{-2} = -\frac{1}{x^2} dx$$

$$-du = \frac{1}{x^2} dx$$

$$\frac{1}{2} \int \cos u du$$

$$u = 2x$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$-\int e^u du = -e^u + C$$

$$-e^{1/x} + C$$

6) $\int \frac{1+x}{1+x^2} dx$

$$= \int \frac{1}{1+x^2} dx + \int \frac{x}{1+x^2} dx$$

$$\int \frac{1}{u} du$$

$$u = 1+x^2$$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$\tan^{-1} x + \frac{1}{2} \ln(1+x^2) + C$$

$$7) \int \frac{4x^2}{(x^3-1)^3} dx \quad u=x^3-1 \quad du=3x^2 dx \cdot \frac{4}{3}$$

$$\frac{4}{3} du = 4x^2 dx$$

$$\frac{4}{3} \int u^{-3} du = -\frac{4}{3} \cdot \frac{1}{2} \cdot u^{-2} + C$$

$$\frac{-2}{3(x^3-1)^2} + C$$

$$11) \int x \sqrt{3x-6} dx$$

$$u=3x-6$$

$$du=3dx$$

$$\frac{1}{3} du = dx$$

$$u+6=3x$$

$$x = \frac{u+6}{3}$$

$$8) \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx \quad u=e^x + e^{-x}$$

$$du = e^x - e^{-x} dx$$

$$\int \frac{1}{u} du = \ln|e^x + e^{-x}| + C$$

$$\ln(e^x + e^{-x}) + C$$

$$\frac{1}{3} \int \left(\frac{u+6}{3}\right) (u^{1/2}) du$$

$$= \frac{1}{9} \int (u+6) u^{1/2} du$$

$$= \frac{1}{9} \int u^{3/2} + 6u^{1/2} du$$

$$\frac{1}{9} \left(\frac{2}{5} u^{5/2} + 6 \cdot \frac{2}{3} u^{3/2} \right) + C$$

$$\frac{2}{45} (3x-6)^{5/2} + \frac{4}{9} (3x-6)^{3/2} + C$$

$$9) \int \frac{(\ln x)^7}{x} dx \quad u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int u^7 du = \frac{(\ln x)^8}{8} + C$$

$$12) \int \frac{4e^{3x}}{3+5e^{3x}} dx \quad u=3+5e^{3x}$$

$$du = 3 \cdot 5e^{3x} dx$$

$$\frac{4}{15} du = 4e^{3x} dx$$

$$\frac{4}{15} \int \frac{1}{u} du =$$

$$\frac{4}{15} \ln|3+5e^{3x}| + C$$

$$10) \int \frac{\sec(\ln x) \tan(\ln x)}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int \sec u \tan u du$$

$$= \sec u + C$$

$$\sec(\ln x) + C$$

$$13) \int x(x+1)^9 dx \quad \begin{array}{l} u=x+1 \\ du=dx \\ x=u-1 \end{array}$$

$$\int (u-1)u^9 du$$

$$= \int (u^{10} - u^9) du = \frac{u^{11}}{11} - \frac{u^{10}}{10} + C$$

$$\frac{(x+1)^{11}}{11} - \frac{(x+1)^{10}}{10} + C$$

$$16) \int (4x-1)^{1/2} dx$$

$$\begin{array}{l} u=4x-1 \\ du=4dx \\ \frac{1}{4}du=dx \end{array}$$

$$\frac{1}{4} \int u^{1/2} du =$$

$$\frac{1}{4} \cdot \frac{2}{3} (4x-1)^{3/2} + C$$

$$\frac{1}{6} (4x-1)^{3/2} + C$$

$$14) \int \sin(2x-4) dx \quad \begin{array}{l} u=2x-4 \\ du=2dx \\ \frac{1}{2}du=dx \end{array}$$

$$\frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos(2x-4) + C$$

$$17) \int x \cos(x^2) dx$$

$$\begin{array}{l} u=x^2 \\ du=2x dx \\ \frac{1}{2}du=x dx \end{array}$$

$$\frac{1}{2} \int \cos u du$$

$$= \frac{1}{2} \sin(x^2) + C$$

$$15) \int \frac{x^3}{(x^4+1)^4} dx \quad \begin{array}{l} u=x^4+1 \\ du=4x^3 dx \\ \frac{1}{4}du=x^3 dx \end{array}$$

$$\frac{1}{4} \int u^{-4} du$$

$$\frac{1}{4} \cdot \frac{1}{-3} (x^4+1)^{-3}$$

$$-\frac{1}{12} (x^4+1)^{-3} + C$$

$$-\frac{1}{12(x^4+1)^3} + C$$

$$18) \int \sin^5 x \cos x dx$$

$$\begin{array}{l} u=\sin x \\ du=\cos x dx \end{array}$$

$$\int u^5 du =$$

$$\frac{u^6}{6} + C = \frac{\sin^6 x}{6} + C$$

$$19) \int \sec^2 x \tan^4 x dx$$

$$\begin{array}{l} u=\tan x \\ du=\sec^2 x dx \end{array}$$

$$\int u^4 du =$$

$$\frac{\tan^5 x}{5} + C$$

$$20) \int \frac{x^3}{(5-x^4)^2} dx \quad u=5-x^4 \quad x=u^{-1/4}$$

$$du = -4x^3 dx \quad u=x+2$$

$$-\frac{1}{4} du = x^3 dx \quad du = dx$$

$$-\frac{1}{4} \int u^{-2} du$$

$$-\frac{1}{4} \cdot \frac{1}{(5-x^4)} + C$$

$$23) \int x(x+2)^{1/2} dx$$

$$\int (u-2)u^{1/2} du$$

$$= \int u^{3/2} - 2u^{1/2} du$$

$$\frac{2}{5} u^{5/2} - 2 \cdot \frac{2}{3} u^{3/2}$$

$$\frac{2}{5} (x+2)^{5/2} - \frac{4}{3} (x+2)^{3/2} + C$$

$$21) \int \frac{x-1}{x} dx$$

$$\int \frac{x dx}{x} - \int \frac{1 dx}{x}$$

$$= \int 1 dx - \int \frac{1}{x} dx$$

$$= x - \ln x + C$$

$$22) \int \frac{5-4x^3+2x^6}{x^6} dx$$

$$\int 5x^{-6} - 4 \int x^{-3} + \int 2 dx$$

$$\frac{5x^{-5}}{-5} - \frac{4 \cdot \frac{1}{2} x^{-2}}{-2} + 2x + C$$

$$-\frac{1}{x^5} + \frac{2}{x^2} + 2x + C$$

$$24) \int \cot x dx$$

$$= \int \frac{\cos x}{\sin x} dx \quad u = \sin x$$

$$du = \cos x dx$$

$$\int \frac{1}{u} du =$$

$$\ln |\sin x| + C$$