Integration Review:

① $\int_1^3 x^2 \ln x\,dx$ $\quad u=\ln x \quad v=\frac{x^4}{4}$ $\quad du=\frac{1}{x}dx \quad dv=x^3dx$ ④ omit

D

$\frac{x^4}{4}\cdot\ln x - \int\frac{x^4}{4}\cdot\frac{1}{x}\,dx$

$\frac{x^4}{4}\cdot\ln x - \frac{1}{4}\int x^3dx$

$\frac{x^4}{4}\ln x - \frac{1}{4}\cdot\frac{1}{4}x^4 \Big|_1^3$

$\left(\frac{81}{4}\ln 3 - \frac{81}{16}\right)-\left(-\frac{1}{16}\right)$

$\frac{81}{4}\ln 3 - \frac{80}{16}$

$\frac{81}{4}\ln 3 - 5$

② $\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx$ $\quad u=x \quad v=\tan x$ $\quad du=dx \quad dv=\sec^2x\,dx$

D

$x\tan x - \int \tan x\,dx$

$x\tan x + \ln|\cos x| \Big|_0^{\pi/4}$

$\left(\frac{\pi}{4}\cdot\tan\frac{\pi}{4} + \ln\left|\cos\frac{\pi}{4}\right|\right) - 0$

$\frac{\pi}{4} + \ln\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{4} + \frac{1}{2}\ln 2 - \ln 2$

$\frac{\pi}{4} - \frac{1}{2}\ln 2$

⑤ $\int_2^{\infty}\frac{1}{x^2-x}\,dx = \lim_{b\to\infty}\int_2^b \frac{1}{x(x-1)}\,dx$

B

$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$1 = A(x-1) + Bx$

$x=1:\ 1 = B(1) \quad B=1$

$x=0:\ 1 = -1A \quad A=-1$

$\lim_{b\to\infty}\int_2^b\left(-\frac{1}{x} + \frac{1}{x-1}\right)dx$

$\lim_{b\to\infty} -\ln|x| + \ln|x-1| \Big|_2^b$

$\lim_{b\to\infty}\left(-\ln|b| + \ln|b-1|\right) - \left(-\ln 2 + \underbrace{\cancel{\ln 1}}_{0}\right)$

$\lim_{b\to\infty} \ln\left|\underbrace{\cancel{\frac{b-1}{b}}}_{0}\right| + \ln 2$

$= \ln 2$

③ $\int_1^2 \frac{x^3}{\sqrt{x^2-1}}\,dx$ $\quad u=x^2-1 \quad x^2=u+1$ $\quad du=2x\,dx$ $\quad \frac{1}{2}du = x\,dx$ $\quad u(1)=0 \quad u(2)=3$

C

$\frac{1}{2}\int\frac{u+1}{u^{1/2}}\,du \to \frac{1}{2}\int\left(u^{1/2}+u^{-1/2}\right)du$

$\frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) = \frac{1}{3}u^{3/2} + u^{1/2}\Big|_0^3$

$\frac{1}{3}\sqrt{27} + \sqrt{3} = \frac{1}{3}\cdot 3\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

6. $\lim\limits_{n\to\infty}\int_0^2 x^{-1/n}\,dx$, $n>1$

C

$\lim\limits_{a\to0^+}\int_a^2 x^{-1/n}\,dx$

$\lim\limits_{a\to0^+} \dfrac{x^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\Bigg|_a^2$

$\lim\limits_{a\to0^+}\left(\dfrac{2^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}-\dfrac{a^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right)$

$\lim\limits_{a\to0^+}\left(\dfrac{2^{-\frac{1}{n}+1}-a^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right)$

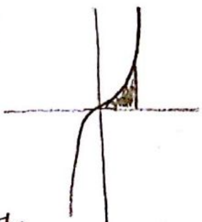
$\lim\limits_{n\to\infty}\dfrac{2^{-\frac{1}{n}+1}-(\text{small})^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}$

$=\dfrac{2^{0+1}-(\text{small})^{0+1}}{0+1}$

$=\dfrac{2-0}{1}=2$

7.

B



$\int_0^{\pi/4}(\tan x)^2\,dx=\int_0^{\pi/4}\dfrac{\sin^2 x}{\cos^2 x}\,dx$

$\int_0^{\pi/4}\dfrac{1-\cos^2 x}{\cos^2 x}\,dx=\int_0^{\pi/4}\left(\dfrac{1}{\cos^2 x}-1\right)dx$

$=\int_0^{\pi/4}(\sec^2 x-1)\,dx=\tan x - x\Big|_0^{\pi/4}$

$\left(\tan\frac{\pi}{4}-\frac{\pi}{4}\right)-0=1-\frac{\pi}{4}$

8. $\int_1^2 x\cdot\dfrac{1}{\sqrt{x^2-1}}\,dx$

A

$u=x^2-1$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\frac{1}{2}\int u^{-1/2}du=\frac{1}{2}\cdot 2u^{1/2}$

$\sqrt{u}\rightarrow\sqrt{x^2-1}\Big|_1^2$

$\sqrt{3}-0=\sqrt{3}$

9. $\int x^3 f''(x^2)\,dx$

B

$u=x^2$

$du=2x\,dx$

$v=\frac{1}{2}f'(x^2)$

$dv=f''(x^2)\cdot x\,dx$

$\frac{1}{2}x^2\cdot f'(x^2)-2\cdot\frac{1}{2}\int f'(x^2)\cdot x\,dx$

$\frac{1}{2}x^2 f'(x^2)-\int f'(x^2)\cdot x\,dx$

$u=x^2$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\frac{1}{2}x^2 f'(x^2)-\frac{1}{2}\int f'(u)\,du$

$\frac{1}{2}x^2 f'(x^2)-\frac{1}{2}(f(x^2))+C$

$\frac{1}{2}\left(x^2 f'(x^2)-f(x^2)\right)+C$

(10) $\int_{-2}^{0} \frac{1}{(x+3)(x-2)}dx$

C

$\frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$1 = A(x-2) + B(x+3)$

$x=2: 1 = 5B \quad B = \frac{1}{5}$

$x=-3: 1 = -5A \quad A = -\frac{1}{5}$

$\frac{1}{5}\int \frac{1}{x-2}dx - \frac{1}{5}\int \frac{1}{x+3}dx$

$\frac{1}{5}(\ln|x-2| - \ln|x+3|)\Big|_{-2}^{0}$

$\frac{1}{5}[(\ln 2 - \ln 3) - (\ln 4 - \underbrace{\cancel{\ln 1}}_{0})]$

$\frac{1}{5}(\ln 2 - \ln 3 - \ln 4)$

$\frac{1}{5}\ln\frac{1}{6}$ or $-\frac{1}{5}\ln 6$

(11) $2\int_{\pi/4}^{\pi/2} \csc^2 x\, dx$

E

$-2\cot x \Big|_{\pi/4}^{\pi/2}$

$-2(\cot\frac{\pi}{2} - \cot\frac{\pi}{4})$

$= -2(0-1)$

$= 2$

(12) $\int \frac{1}{5(1+\cos x)} \cdot \frac{(1-\cos x)}{(1-\cos x)} dx$

C

$\frac{1}{5}\int \frac{1-\cos x}{1-\cos^2 x}dx$

$\frac{1}{5}\int \frac{1-\cos x}{\sin^2 x}dx$

$\frac{1}{5}\left(\int \frac{1}{\sin^2 x}dx - \int \frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}dx\right)$

$\frac{1}{5}\left(\int \csc^2 x\, dx - \int \cot x \csc x\, dx\right)$

$\frac{1}{5}(-\cot x + \csc x) + C$

(13) $\int_1^\infty \frac{1}{x(\ln x+1)^2}dx \rightarrow \lim_{b\to\infty}\int_1^b \frac{1}{x(\ln x+1)^2}dx$

B

$u = \ln x + 1$

$du = \frac{1}{x}dx$

$\lim_{b\to\infty}\int u^{-2}du$

$\lim_{b\to\infty} \frac{-1}{\ln x+1}\Big|_1^b = \lim_{b\to\infty}\left(\frac{-1}{\ln b+1} - \left(\frac{-1}{1}\right)\right)$

$= 1$

(14) $\int_0^2 \frac{1}{\sqrt[5]{1-x}}dx$

A

$u = 1-x$

$du = -dx$

$-du = dx$

$\lim_{b\to 1^-} -\int_0^b u^{-1/5}du + \lim_{a\to 1^+} -\int_a^2 u^{-1/5}du$

$\lim_{b\to 1^-}\left(\frac{-5}{4}(1-b)^{4/5} + \frac{5}{4}\right) + \lim_{a\to 1^+}\left(\frac{-5}{4}(-1)^{4/5} + \frac{5}{4}(1-a)^{4/5}\right)$

$(0 + \frac{5}{4}) + (\frac{-5}{4} + 0) = 0$

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15. $xy^2 + \ln x = y + 6$

E

$x \cdot 2y\frac{dy}{dx} + y^2 + \frac{1}{x} = \frac{dy}{dx}$

$y^2 + \frac{1}{x} = \frac{dy}{dx} - 2xy\frac{dy}{dx}$

$\frac{y^2 + \frac{1}{x}}{1 - 2xy} = \frac{dy}{dx}$

$\frac{dy}{dx}\Big|_{(1,3)} = \frac{9+1}{1-2(3)} = \frac{10}{-5} = -2$

$y - 3 = -2(x-1)$

$y - 3 = -2x + 2$

$y = -2x + 5$

16. $\ln y + \ln x = -x^2 + 1$

C

$\frac{1}{y}\frac{dy}{dx} + \frac{1}{x} = -2x$

$\frac{dy}{dx} = \left(-2x - \frac{1}{x}\right)y$

$\ln y + \ln x = -x^2 + 1$

$\ln y = -\ln x - x^2 + 1$

$y = e^{-\ln x - x^2 + 1}$

$y = e^{-\ln x} \cdot e^{-x^2} \cdot e^1$

$y = e^{\ln(\frac{1}{x})} \cdot e^{-x^2} \cdot e$

$y = \frac{1}{x} \cdot \frac{1}{e^{x^2}} \cdot e$

$y = \frac{e}{xe^{x^2}}$

$\frac{dy}{dx} = \left(-2x - \frac{1}{x}\right)\left(\frac{e}{xe^{x^2}}\right)$

$\lim_{x \to \infty} y' = (-\infty - 0)(0)$

indeterminate

17. $\lim_{x \to \infty} x^{1/\ln x}$

D

$\ln y = \lim_{x \to \infty} \frac{1}{\ln x} \cdot \ln x$

$\ln y = 1$

$e^1 = y$

$y = e$

18. $\int_{-2}^{2} \arctan x \, dx$

A

$u = \arctan x \quad v = x$

$du = \frac{1}{1+x^2}dx \quad dv = dx$

$x\arctan x - \int \frac{x}{1+x^2}dx \quad u = 1+x^2, \; du = 2x\,dx, \; \frac{1}{2}du = x\,dx$

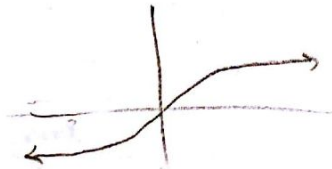
$x\arctan x - \frac{1}{2}\int \frac{1}{u}du$

$x\arctan x - \frac{1}{2}\ln|1+x^2|\Big|_{-2}^{2}$

$\left(2\arctan 2 - \cancel{\tfrac{1}{2}\ln 5}\right) - \left(-2\arctan(2) - \cancel{\tfrac{1}{2}\ln 5}\right)$

$2\arctan 2 - (2\arctan 2) = 0$

$2\arctan 2 + 2\arctan(-2)$



$\lim_{x \to \infty}\left(\frac{-2\cancel{x}\cancel{e}}{\cancel{x}\cancel{e}x^2} - \frac{e}{x^2e^{x^2}}\right)$

$\lim_{x \to \infty}\left(\frac{-2}{x^2} - \frac{e}{x^2e^{x^2}}\right)$

$0 - 0$

0

(19)

E

x	y	dy/dx
1	2	1
1.1	2.1	10/11
1.2	2.191	

$$\frac{dy}{dx} = \frac{y-1}{x^2}$$

$$y = 2 + .1(1)$$

$$y = 2.1 + .1(10/11)$$

(20)

D

$$\lim_{x \to 2} \frac{2x^2-8}{x-2} \to \frac{2(x^2-4)}{x-2} \to \frac{2(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} 2(x+2) = 8$$