Integration Review
Calculus AB

Name: ____________________________

Date:____________________________

I. Given $y = x\sin x$ on the interval [0, 1]. Using 4 subinterval find

a) Left hand Rectangular Approximation

b) Right hand Rectangular approximation

c) Find Midpoint Rectangular approximation

II. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x)dx = -4 \qquad \int_1^5 f(x)dx = 6 \qquad \int_1^5 g(x)dx = 8$$

Find

a) $\int_2^2 f(x)dx$

c) $\int_1^2 3f(x)dx$

e) $\int_1^5 [f(x)+g(x)]dx$

b) $\int_1^5 g(x)dx$

d) $\int_2^5 f(x)dx$

f) $\int_1^5 [4f(x)-g(x)]dx$

III. Suppose that f and h are continuous functions such that

$$\int_1^9 f(x)dx = -1 \qquad \int_7^9 f(x)dx = 5 \qquad \int_7^9 h(x)dx = 4$$

Find

a) $\int_1^9 -2f(x)dx$

c) $\int_7^9 [2f(x)-h(x)]dx$

e) $\int_1^7 f(x)dx$

b) $\int_7^9 [f(x)+h(x)]dx$

d) $\int_9^1 f(x)dx$

f) $\int_9^7 [h(x)-f(x)]dx$

IV. Evaluate each integral

a) $\int_3^1 7dx$

c) $\int_2^3 \frac{x^2+2x^3}{x^2}dx$

e) $\int_0^{\pi} (e^x+\cos x)dx$

b) $\int_0^9 5\sqrt{x}dx$

d) $\int_0^{\sqrt{2}} (t-\sqrt{2})dt$

f) $\int_0^4 \sqrt{x}(x+1)dx$

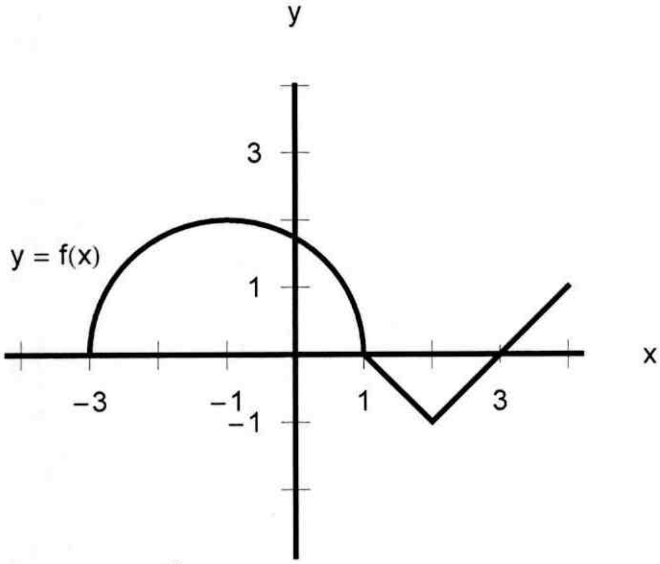
V. Find dy/dx of the following

a. $y = \int_0^x \sqrt{1+t^2}\,dt$

b. $y = \int_0^{\sqrt{x}} \sin(t^2)dt$

c. $y = \int_{x^2}^0 \cos t\,dt$

5. The graph of a function f consists of a semicircle and two line segments as shown below.



Let $g(x) = \int_1^x f(t)\,dt$

(a) Find g(1)

(b) Find g(3)

(c) Find g(-1)

(d) Find all values of x on the open interval (-3, 4) at which g has a local minimum.

(e) Write an equation for the line tangent to the graph of g at x = -1.

(f) Find the x-coordinate of each point of inflection of the graph of g on the open interval (-3, 4).

(g) Find the range of g.

Review of Integration
Calculus AP

Name: ______________________
Date: ______________________

1. Rocket A has positive velocity v(t) after being launched upward from an initial height of 0 feet at time t = 0 seconds. The velocity of the rocket is recorded for selected values of t over the interval [0, 80] seconds as shown in the table below. (2006 AB4/BC4)

t (sec)	0	10	20	30	40	50	60	70	80
v(t) (f/s)	5	`14	22	29	35	40	44	47	49

a) Using correct units, explain the meaning of $\int_{10}^{70} v(t)dt$in terms of the rocket's flight.

b) Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)dt$

2. A metal wire of length 8 centimeters (cm) is heated at one end, The table below gives selected values of the Temperature T(x) in degrees Celsius, of the wire x cm from the heated end. The function T is decreasing and twice differentiable. (2005 AB3/BC3)

Distance, x (cm)	0	1	5	6	8
Temperature T(x) ^{0}C	100	93	70	62	55

a. Estimate T'(7). Show the work that leads to your answer. Indicate units of measure.
b. Write an integral expression in term of T(x) for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with four subintervals indicated by the data in the table. Indicate units of measure.

3. Use trapezoids to estimate the definite integral of $f(x) = -100x^2 + 90x + 14$ from x = 0.1 to x =1. Use 9 increments.

4. The velocity (in mph) of a Piper Club aircraft traveling due west is recorded every minute during the first 10 min after takeoff. Use the Trapezoidal Rule to estimate the distance traveled after 10 minutes.

t	0	1	2	3	4	5	6	7	8	9	10
v(t)	0	50	60	80	90	100	95	85	80	75	85

5. Approximate the area bounded by $f(x) = 4 - x^2$ and the x-axis using
   a) Left-hand rectangles and four equal subdivisions
   b) Right-hand rectangles and four equal subdivisions
   c) Midpoint Rectangles and four equal subdivisions
   d) Approximate this area using a Trapezoidal Rule and four equal subdivisions
   e) Verify that that the Left-hand approximation and the Right hand approximation give the same result as the Trapezoidal rule

Review of Integration
Calculus AP

Name: ____________________
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6. The table below shows the velocity of a remote-controlled toy car as it traveled down a hallway for 10 seconds.

t	0	1	2	3	4	5	6	7	8	9	10
v(t) in/sec	0	6	10	16	14	12	18	22	12	4	2

Estimate the distance traveled by the car using 10 subintervals of length 1 and the following methods

a) Left-hand rectangles
b) Right-hand rectangles

7. The table shows the rate in liters/minute at which water leaked out of a container.

Time (min)	0	1.2	2.3	3.8	5.4
Rate (liters/min)	5.6	4.3	3.1	2.2	1.5

A right hand Riemann sum is computed using the four subintervals indicated by the data in the table. This Riemann sum estimates the total amount of water that has leaked out of the container. What is the estimate?

A) 12.70 *l*   B) 14.27 *l*   C)16.7 *l*   D) 16.95 *l*   E) 19.62 *l*

8. The temperature, in degrees Celsius (^{0}C) of a turkey in an oven is a continuous function of time t. Some values of this function are given in the table.

Time (min)	0	5	10	15	20	25
Temperature °C	24	76	106	124	135	141

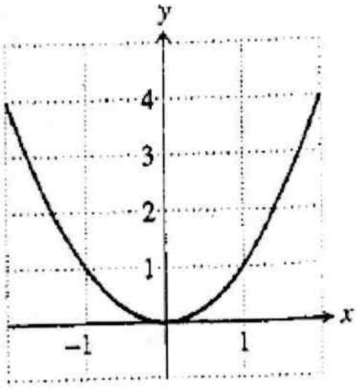
Approximate the average temperature (in degrees Celsius) of the turkey over the interval [0, 25] using a left-hand Riemann sum with subintervals of length 5 minutes.

9. The table below shows the velocity of a remote-controlled toy car as it traveled down a hallway for 10 seconds.

Time (sec)	0	1	2	3	4	5	6	7	8	9	10
Velocity (in/sec)	0	6	10	16	14	12	18	22	12	4	2

Using the Trapezoidal Rule, estimate the distance traveled by the car. Use 10 subintervals of length 1.

10. The graph of y = *f*(x) is shown. Use the Trapezoidal Rule with n = 4 to estimate the area bounded by the graph of y = *f*(x) and the x-axis on the intervals [-2, 2].



Review of Integration
Calculus AP

Name: ____________________

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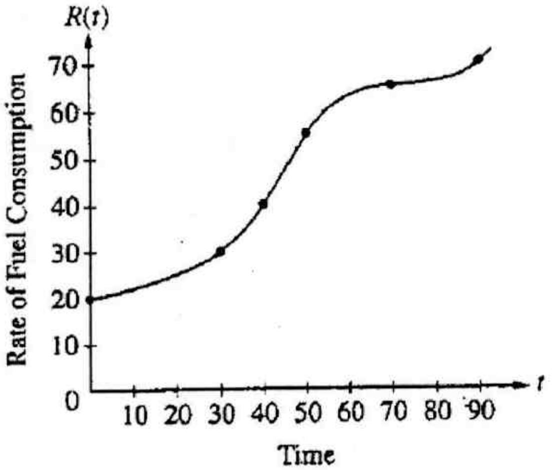
11. The function f is continuous on the closed interval [1, 7] and has the values given in the table.

x	1	4	6	7
f(x)	10	20	40	30

Using subintervals [1, 4], [4, 6], [6, 7], what is the trapezoidal approximation of $\int_a^b f(x)dx$?

a) 110   b) 120   c) 130   d) 140   e) 150

12. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of R(t) for the time interval [0, 90] minutes are shown below. (2003 AB3)



t (minutes)	R(t) (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

a) Use the data from the table to find an approximation for R'(45). Show the computations that led to your answer. Indicate units of measure.

b) Approximate the value of $\int_0^{90} R(t)dt$ using a left Riemann sum with five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)dt$? Explain your reasoning.

13. On a ship at sea, it is easier to measure how fast you are going than it is to measure how far you have gone. Suppose you are the navigator aboard a supertanker. The velocity of the ship is measured every 15 min and recorded in the table below. Estimate the distance the ship has gone between 7:30 pm and 9:15 pm

Time	mi/hr	Time	mi/hr
7:30	28	8:30	7
7:45	25	8:45	10
8:00	20	9:00	21
8:15	22	9:15	26

Review of Integration
Calculus AP

Name: ______________________

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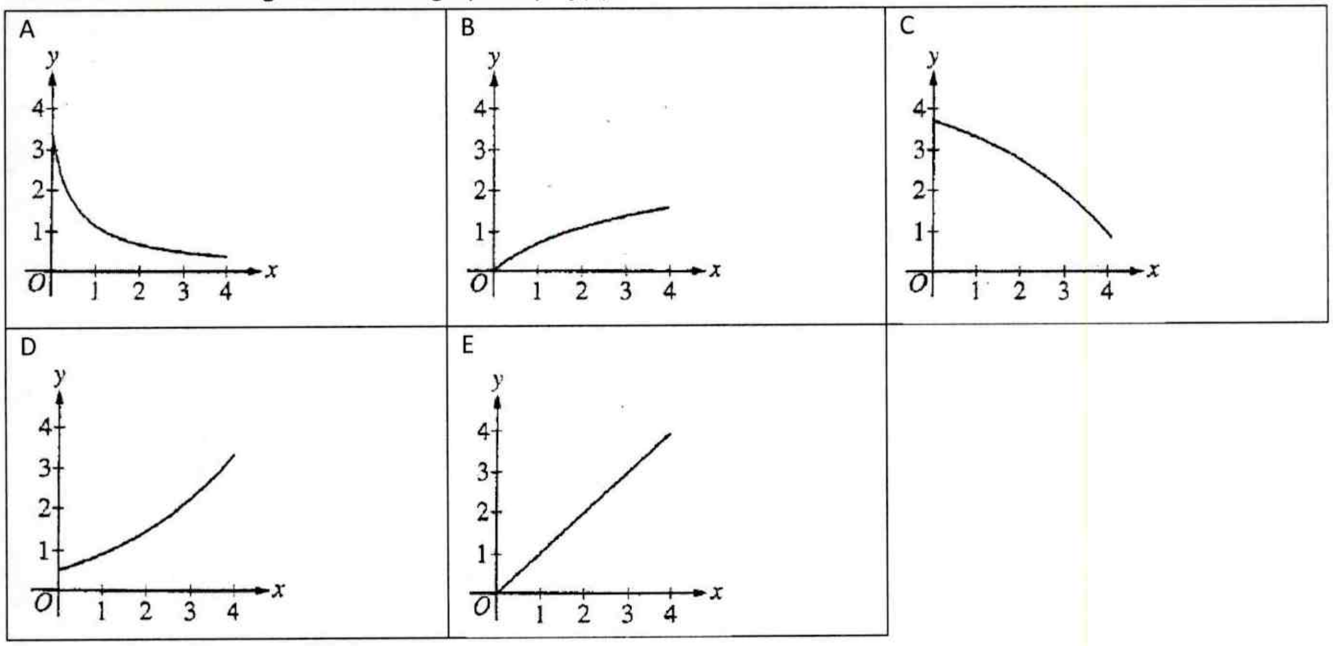
14. If a trapezoid sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of y = *f*(x)?



15. Given the following function: $y = 2x - x^2$ and the interval [1, 2], find the area using the following:
    a. 4 Inscribed rectangles
    b. 4 circumscribed rectangles
    c. Trapezoidal Rule with n = 4
    d. Midpoint Formula with n = 4
    e. Find the exact area under the curve

16. Evaluate the following
    a. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} cosxdx$
    b. $\int_1^9 2x\sqrt{x}$
    c. $\int_0^1 (x^4 - 5x^3 + 3x^2 - 4x - 6)dx$
    d. $\int_{-4}^{4} |x|dx$
    e. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sinxdx$
    f. $\int_4^9 \frac{2+x}{2\sqrt{x}}dx$

17. Find the following
    a. $\int \left(x - \frac{1}{2x}\right)^2$
    b. $\int \frac{2x+1}{2x}dx$
    c. $\int \frac{x^3-x-1}{x^2}dx$
    d. $\int \sqrt{x}(\sqrt{x} - 1)dx$
    e. $\int \frac{e^{2x}+1}{e^x}dx$
    f. $\int_{-3}^{4} |x + 1|dx$

IF U = $\frac{1}{2}$(LOOP)^2, THEN DU = (?).

Choose a function to substitute for u.

1) $\int 2x(x^2+1)^5\,dx$	2) $\int \frac{2x}{x^2+1}\,dx$	3) $\int 2x\sqrt{x^2+1}\,dx$	4) $\int 3x^2(x^3+1)^5\,dx$
5) $\int \sin^3(x)\cos(x)\,dx$	6) $\int \frac{\cos(x)}{\sin(x)}\,dx$	7) $-\int \sin(x)e^{\cos(x)}\,dx$	8) $-\int \frac{\sin(x)}{\cos^2(x)}\,dx$
9) $\int \tan^3(x)\sec^2(x)\,dx$	10) $\int \cos(x)\sqrt{1+\sin(x)}\,dx$		

Let u = (?).

A. $u = \sin^3(x)$	D. $u = 1+\sin(x)$	L. $u = \cos(x)$	M. $u = x^2$	O. $u = x^2 + 1$
P. $u = \sin(x)$	R. $u = \tan^3(x)$	S. $u = e^x$	(. $u = \tan(x)$	). $u = x^3 + 1$

8	1	1	6

10	9	7	2	3	5	4

WHAT POLITICAL MOVEMENT AIMS TO PREVENT THE TEACHING OF CALCULUS IN HIGH SCHOOLS?

For the indefinite integrals **1) – 10)** above, **1a) – 10a)** du= (?).

A. $du = 3x^2dx$	B. $du = \tan(x)dx$	E. $du = 2xdx$	I. $du = -\sin(x)dx$
K. $du = \sec(x)dx$	M. $du = \sec^2(x)dx$	P. $du = \sin(x)dx$	R. $du = \tan^3(x)dx$
S. $du = e^xdx$	T. $du = \cos(x)dx$	U. $du = -\cos(x)dx$	V. $du = (x^3 + 1)dx$

For the indefinite integrals **1) – 10)** above, **1b) – 10b)** give the indefinite integral.

A. $\frac{1}{6}(x^2+1)^6 + k$	D. $\frac{1}{6}(x^3+1)^6 + k$	E. $\frac{2}{3}(x^2+1)^{3/2} + k$	H. $\frac{1}{4}\sin^4(x) + k$
M. $\frac{1}{4}\tan^4(x) + k$	N. $-\sec(x) + k$	O. $e^{\cos(x)} + k$	P. $\sec(x) + k$
P. $\sec^3(x) + k$	R. $\frac{2}{3}(\sin x + 1)^{3/2} + k$	T. $\ln(x^2+1) + k$	V. $\ln\lvert\sin(x)\rvert + k$

5a	5b	1a

4a	8b	10a	7a

-

4b	3a	10b	8a	6b	1b	2b	8a	6b	1a

9a	7b	6b	2a	9b	3b	8b	6a

AP Calculus AB
Integration Practice Worksheet

1) $\int \sec^2 x \tan x dx$	2) $\int (x^2+1)\sqrt{1+3x+x^3}dx$
3) $\int \frac{x^2}{\sqrt{5+x^3}}dx$	4) $\int x\cos(x^2)\sin^3(x^2)dx$
5) $\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}dt$	6) $\int_0^{\pi/2} \sin^5 t \cos t dt$
7) $\displaystyle\int \sec 2\theta \tan 2\theta \ d\theta$	8) $\displaystyle\int \cos^4 \theta \sin \theta \ d\theta$

1976 AB6

(a) Given $5x^3 + 40 = \int_c^x f(t)\,dt$.

(i) Find $f(x)$.

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt$, find $F'(x)$.

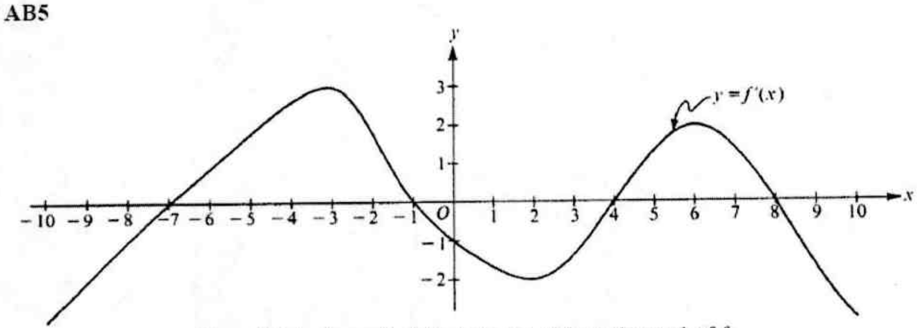
1977 BC7

Let $F(x) = \int_0^x \frac{1}{1+t^4}\,dt$ for all real numbers x.

(a) Find $F(0)$.

(b) Find $F'(1)$.

1989 AB5



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

(c) For value of x is the graph of f concave downward?

Water is draining out of a tank at a variable rate as given by the chart below.

t	R(t) Gal/min
0	0
5	5
10	20
20	30
30	15
35	0

a. Approximate the volume of water that has leaked from the tank for 0≤t≤35 using a Riemann sum with a right-hand endpoint for the five unequal intervals indicated by the data in the chart.

b. Approximate the average # of Gallons in the tank using your approximation in part a.

c. Use the data from the table to approximate R'(25). Show the computation that leads to your answer.

d. If the rate of the leak is modeled by $Q(t) = 16.78\sin(0.15x - 1.25) + 14.6$, Find Q'(25). Using appropriate units, explain the meaning of your answer.

e. Use the function Q defined in part d to find the average value, in Gallons, of Q(t) over the time interval [5, 25]

1991 AB1

Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x) = 24x - 18$

(ii) $f'(1) = -6$

(iii) $f(2) = 0$

(a) Find each x such that the line tangent to the graph of f at $(x, f(x))$ is horizontal.

(b) Write an expression for $f(x)$.

(c) Find the average value of f on the interval $1 \le x \le 3$.

1994 AB 1

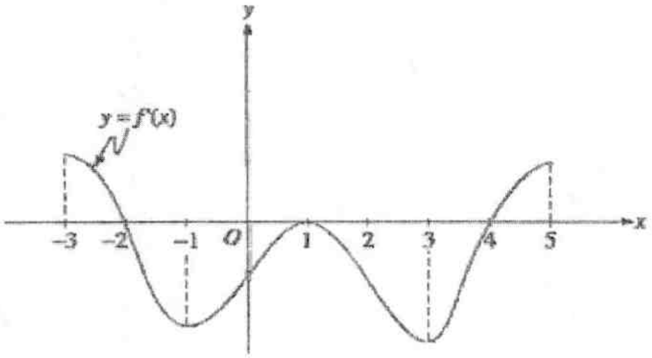
Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

(a) Write an equation of the line tangent to the graph of f at the point $(2, -28)$.

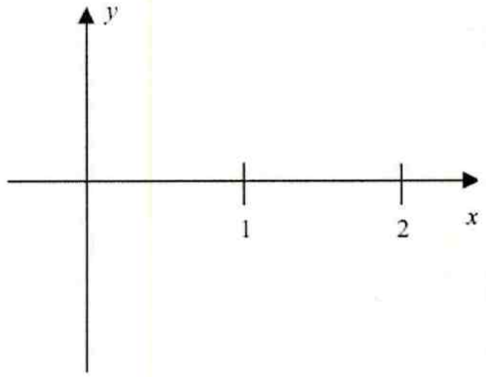
(b) Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

(c) Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

1996 AB1



Note: This is the graph of the derivative of f, not the graph of f.



The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Why?

(b) For what values of x does f have a relative minimum? Why?

(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

1970 AB3/BC2

Consider the function f given by $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}}$ on the interval $-8 \le x \le 8$.

(a) Find the coordinates of all points at which the tangent to the curve is a horizontal line.

(b) Find the coordinates of all points at which the tangent to the curve is a vertical line.

(c) Find the coordinates of all points at which the absolute maximum and absolute minimum occur.

(d) For what values of x is this function concave down?

(e) On the axes provided, sketch the graph of the function on this interval.