Integration Review (All Methods)

Name: KEY
Date: ____________

1. $\int_0^\infty \frac{-4}{x^2+11x+30}dx$	2. $\int \frac{2x}{x^2-3x-10}dx$
3. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx$	4. $\int_1^\infty xe^{-5x}dx$
5. $\lim_{x\to\infty}\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x$	6. $\int \frac{x-x^2}{2\sqrt[3]{x}}dx$
7. $\int \frac{dx}{(x+3)(x-2)}$	8. $\int ln(x^{157})dx$
9. $\int \frac{x^2}{(1-x^3)^2}dx$	10. $\int_0^9 \frac{dx}{(x-9)^{\frac{2}{3}}}$
11. $\lim_{x\to 0}\frac{\tan px}{\tan qx}$ p and q are constants	12. $\int_{-2}^{2}\frac{1}{x}dx$
13. $\lim_{x\to 1}\left(\frac{\ln x^2}{x^2-1}\right)$	14. $\int x\sqrt{x-5}dx$
15. $\int x^2 sin2x$dx	16. $\lim_{x\to\infty} xe^{-x^2}$
17. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$	18. $\int \frac{x}{\sqrt{1-x^2}}dx$

Integration Review

1) $\int_0^\infty \frac{-4}{x^2+11x+30}dx$ $\quad \frac{-4}{(x+6)(x+5)} = \frac{A}{x+6} + \frac{B}{x+5}$

$(x+6)(x+5)$

$-4 = A(x+5) + B(x+6)$

$x = -5:\ -4 = B(1) \quad B = -4$

$x = -6:\ -4 = A(-1) \quad A = 4$

$$\int_0^\infty \frac{4}{x+6} + \frac{-4}{x+5}dx = \lim_{b\to\infty} \int_0^b \frac{4}{x+6} + \frac{-4}{x+5}dx$$

$$= \lim_{b\to\infty} \left[4\ln(x+6) + {}^-4\ln(x+5)\right]\Big|_0^b$$

$$= \lim_{b\to\infty} \left[4\ln(b+6) - 4\ln(b+5) - (4\ln 6 - 4\ln 5)\right]$$

2) $\int \frac{2x}{x^2-3x-10}dx$ $\quad \frac{2x}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$

$2x = A(x+2) + B(x-5)$

$x = -2:\ -4 = B(-7) \quad B = \frac{4}{7}$

$x = 5:\ 10 = A(7) \quad A = \frac{10}{7}$

$$\int \frac{\frac{10}{7}}{x-5} + \frac{\frac{4}{7}}{x+2}dx = \frac{10}{7}\ln|x-5| + \frac{4}{7}\ln|x+2| + C$$

$$\boxed{\ln\frac{|(x-5)^{10}(x+2)^4|}{7} + C}$$

3) $$\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$$

$u = 1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$2\int \frac{1}{u^2}\,du = 2\int u^{-2}\,du = \frac{-2}{u} = \boxed{\frac{-2}{1+\sqrt{x}} + C}$$

4) $$\int_1^{\infty} xe^{-5x}\,dx = \lim_{b\to\infty}\int_1^{b} xe^{-5x}\,dx$$

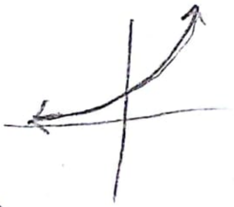
u	dv
$+x$	e^{-5x}
-1	$-\frac{1}{5}e^{-5x}$
0	$\frac{1}{25}e^{-5x}$

$$\lim_{b\to\infty}\left(-\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x}\right)\Big|_1^b$$

$$\lim_{b\to\infty}\left(-\frac{b}{5e^{5b}} - \frac{1}{25e^{5b}}\right) - \left(\frac{-1(5)}{5e^{5}(5)} - \frac{1}{25e^{5}}\right)$$

$$(0-0) - \left(\frac{-5-1}{25e^{5}}\right)$$

$$\frac{+6}{25e^{5}}$$



$$\rightarrow \boxed{\frac{6e^{-5}}{25}}$$

5) $$\lim_{x\to\infty}\left(1+\frac{3}{x}+\frac{5}{x^2}\right)^x$$

$$\ln y = \lim_{x\to\infty} x\ln(1+3x^{-1}+5x^{-2})$$

$$\ln y = \lim_{x\to\infty}\frac{\ln(1+3x^{-1}+5x^{-2})}{\frac{1}{x}}$$

L'H:

$$\ln y = \lim_{x\to\infty}\frac{\frac{1}{1+3x^{-1}+5x^{-2}}\cdot -3x^{-2}-10x^{-3}}{\frac{-1}{x^2}}$$

$$\frac{1}{1+\frac{3}{x}+\frac{5}{x^2}}\cdot\left(-\frac{3}{x^2}-\frac{10}{x^3}\right) \Big/ \frac{-1}{x^2}$$

$$\frac{1}{1+\frac{3}{x}+\frac{5}{x^2}}\left(\frac{-3x-10}{x^3}\right)$$

$$= \frac{-3x-10}{x^3+3x^2+5x}\cdot -x^2$$

$$= \frac{+3x^3+10x^2}{x^3+3x^2+5x} \quad \frac{\infty}{\infty}$$

L'H: $\frac{9x^2+20x}{3x^2+6x+5}$

L'H: $\frac{18x+20}{6x+6}$

L'H: $\frac{18}{6} = 3$

$\ln y = 3$

$\boxed{e^3}$

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6). $\int \frac{x-x^2}{2\sqrt[3]{x}}dx = \int \frac{x}{2\sqrt[3]{x}}dx - \int \frac{x^2}{2\sqrt[3]{x}}dx$ $\quad 6/3 \quad 1/3$

$= \frac{1}{2}\int x^{2/3}dx - \frac{1}{2}\int x^{5/3}dx$

$\frac{1}{2}\cdot\frac{3}{5}x^{5/3} - \frac{1}{2}\cdot\frac{3}{8}x^{8/3} + C$

$\boxed{\frac{3}{10}x^{5/3} - \frac{3}{16}x^{8/3} + C}$

7) $\int \frac{1\,dx}{(x+3)(x-2)}$ $\quad 1 = \frac{A}{x+3} + \frac{B}{x-2}$

$1 = A(x-2) + B(x+3)$

$x=2:\ 1 = B(5) \quad B = \frac{1}{5}$

$x=-3;\ 1 = A(-5) \quad A = -\frac{1}{5}$

$\int \frac{-\frac{1}{5}}{x+3} + \frac{\frac{1}{5}}{x-2}\,dx = -\frac{1}{5}\ln|x+3| + \frac{1}{5}\ln|x-2| + C$

$\frac{1}{5}\left(\ln|x-2| - \ln|x+3|\right) + C$

$\boxed{\frac{1}{5}\left(\ln\left|\frac{x-2}{x+3}\right|\right) + C}$

8) $\int \ln(x^{157})dx$

$u\ln u - u + C$

$\boxed{x^{157}\ln x^{157} - x^{157} + C}$ $\quad 157\ln x\cdot(x^{157}) - x^{157} + C$

$x^{157}(157\ln x - 1) + C$

$u = \ln(x^{157}) \quad dv = 1\,dx$

$du = \frac{1}{x^{157}}\cdot 157x^{156} \quad v = x$

$x\ln(x^{157}) - 157\int \frac{x^{157}}{x^{157}}dx = \boxed{x\ln(x^{157}) - 157x + C}$

$\int 1\,dx = \boxed{x\cdot 157\ln x - 157x + C}$

9) $\int \frac{x^2}{(1-x^3)^2}dx$ $\quad u=1-x^3$

$du=-3x^2dx$

$-\frac{1}{3}du=x^2dx$

$-\frac{1}{3}\int u^{-2}du = -\frac{1}{3}\cdot -\frac{1}{u} = \frac{1}{3u} = \frac{1}{3(1-x^3)}$

or $\boxed{\frac{-1}{3(x^3-1)}}$

10) $\int_0^9 \frac{dx}{(x-9)^{2/3}} \rightarrow \lim_{b\to 9^-}\int_0^b u^{-2/3}du$ $\quad u=x-9$, $du=dx$

$\lim_{b\to 9^-} 3(x-9)^{1/3}\Big|_0^b = \lim_{b\to 9^-} 3(b-9)^{1/3} - 3(0-9)^{1/3}$

$= 0 - 3(-9)^{1/3}$

$-3(-9)^{1/3}$

11) $\lim_{x\to 0}\frac{\tan px}{\tan qx}$ $\frac{0}{0}$ p and q are constants

$\lim_{x\to 0}\frac{p\sec^2 x}{q\sec^2 x} = \boxed{\frac{p}{q}}$

12) $\int_{-2}^{2}\frac{1}{x}dx =$ discont @ $x=0$

$\int_{-2}^{0}\frac{1}{x}dx + \int_{0}^{2}\frac{1}{x}dx$

$\lim_{t\to 0^-} \ln|x|\Big|_{-2}^{t} + \lim_{t\to 0^+}\ln|x|\Big|_t^2$

$\ln t - \ln 2 + \ln 2 - \ln t$ $\quad$ divergent

$-\infty - \cancel{\ln 2} + \cancel{\ln 2} - (-\infty)$

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13) $\lim_{x \to 1} \left(\frac{\ln x^2}{x^2 - 1} \right) \quad \frac{0}{0}$

L'H: $\dfrac{\frac{1}{x^2} \cdot \cancel{2x}}{\cancel{2x}} = \boxed{1}$

14) $\int x\sqrt{x-5}\,dx \qquad u = x-5 \qquad x = u+5$

$du = dx$

$\int (u+5)\,u^{1/2}\,du = \int u^{3/2} + 5u^{1/2}\,du$

$\frac{2}{5}u^{5/2} + 5 \cdot \frac{2}{3}u^{3/2} + C$

$\frac{2}{5}(x-5)^{5/2} + \frac{10}{3}(x-5)^{3/2} + C$

$-c \; s \; c \; -s$

15) $\int x^2 \sin 2x\,dx$

u	dv
$+\,x^2$	$\sin 2x$
$-\,2x$	$-\frac{1}{2}\cos 2x$
$+\,2$	$-\frac{1}{4}\sin 2x$
0	$\frac{1}{8}\cos 2x$

$-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

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16) $\lim_{x\to\infty} xe^{-x^2} = \lim_{x\to\infty} x \cdot \frac{1}{e^{x^2}}$ $\infty \cdot 0$ ...

$$\lim_{x\to\infty} \frac{\frac{1}{e^{x^2}}}{\frac{1}{x}} \quad \frac{0}{0}$$

L'H: $\lim_{x\to\infty} \frac{-2xe^{-x^2}}{-1x^{-2}} \quad \frac{\frac{-2x}{e^{x^2}}}{\frac{-1}{x^2}} \quad \frac{0}{0}$

L'H: $\frac{-2x(e^{-x^2} \cdot -2x) + e^{-x^2} \cdot -2}{2x^{-3}}$

$$\frac{\frac{4x^2}{e^{x^2}} - \frac{2}{e^{x^2}}}{\frac{2}{x^3}} \quad \frac{4x^2-2}{e^{x^2}} \cdot \frac{x^3}{2}$$

$$\lim_{x\to\infty} \frac{4x^5-2x^3}{\underbrace{2e^{x^2}}_{\text{big}}} = \boxed{0}$$

17) $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

$\ln y = \lim_{n\to\infty} n \ln(1+n^{-1})$ $\quad \infty \cdot 0$

$\frac{\ln(1+n^{-1})}{n^{-1}}$ $\quad \frac{0}{0}$

L'H: $\frac{\frac{1}{1+n^{-1}} \cdot \cancel{-1n^{-2}}}{\cancel{-1n^{-2}}} =$

$\lim_{n\to\infty} \frac{1}{1+\underset{0}{\frac{1}{n}}} = 1$

$\ln y = 1$ $\quad \boxed{y=e}$

18) $\int \frac{x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$-\frac{1}{2}\int u^{-1/2}\,du$

$-\frac{1}{2} \cdot 2u^{1/2}$

$-\sqrt{1-x^2} + C$