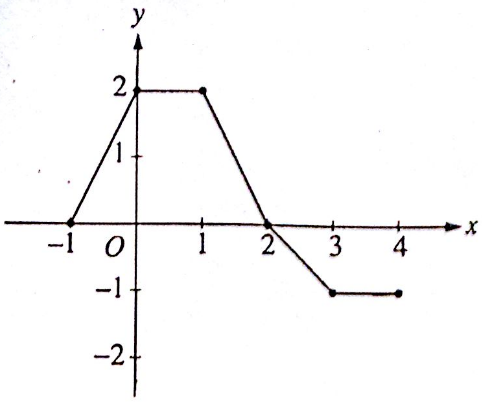

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin\left(t^3\right) dt\right) =$

(A) $-\cos\left(x^6\right)$ (B) $\sin\left(x^3\right)$ (C) $\sin\left(x^6\right)$

(D) $2x\sin\left(x^3\right)$ (E) $2x\sin\left(x^6\right)$

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\,dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

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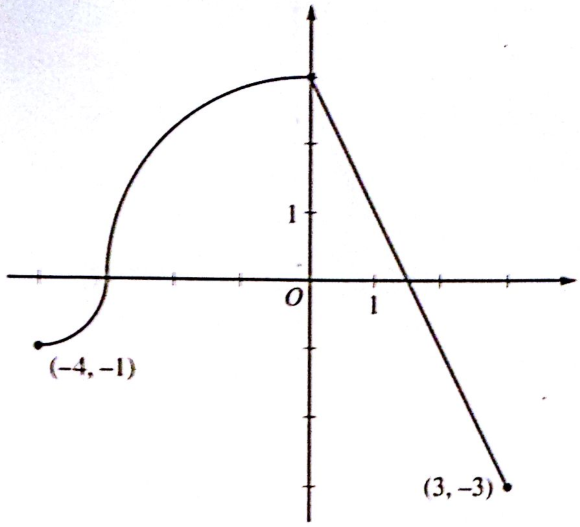

Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles

and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$.
Justify your answer.

(c) Find all values of x on the interval $-4 \le x \le 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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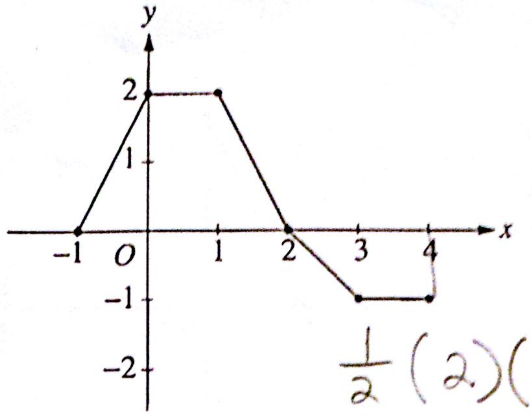
Key



$\frac{1}{2}(1)(2+1) = -\frac{3}{2}$

$\frac{1}{2}(2)(3+1) = 4$

2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

23. $\frac{d}{dx}\left(\int_0^{x^2} \sin(t^3)\,dt\right) =$ $\sin(x^2)^3 \cdot 2x$

(A) $-\cos(x^6)$ (B) $\sin(x^3)$ (C) $\sin(x^6)$

(D) $2x\sin(x^3)$ (E) $2x\sin(x^6)$

15. If $F(x) = \int_0^x \sqrt{t^3+1}\,dt$, then $F'(2) =$ $\sqrt{x^3+1}$

(A) −3 (B) −2 (C) 2 (D) 3 (E) 18

$F(9) - F(1) = \int_1^9 \frac{(\ln x)^3}{x}\,dx$ $u = \ln x$ $du = \frac{1}{x}dx$

88. Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

$(x)dx = F(9) - F(1)$

$F(1) + \int_1^9 f(x)dx = F(9)$

$\int u^3 du = \left.\frac{(\ln x)^4}{4}\right|_1^9 \frac{(\ln 9)^4}{4} - 0$

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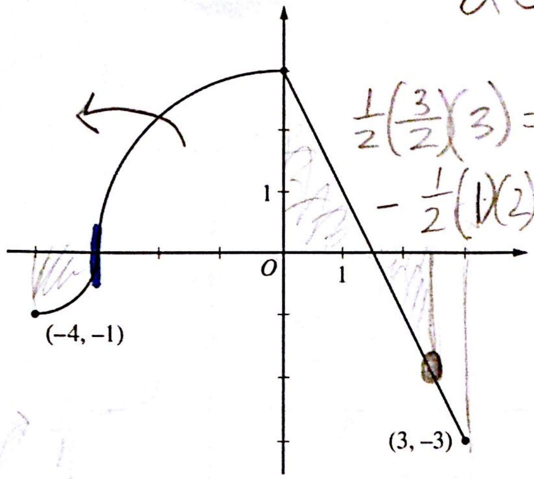
$\frac{1}{4}\pi(3)^2$



Graph of f

$\frac{1}{2}\left(\frac{3}{2}\right)(3) = \frac{9}{4}$

$-\frac{1}{2}(1)(2) = 1$

x	$g(x)$
-4	$-8 + \text{neg}$
2.5	$5 + 5/4 = \frac{25}{4}$
3	$6 + 0 = 6$

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and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(3) (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(3) (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$.
Justify your answer.

(1) (c) Find all values of x on the interval $-4 \le x \le 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(2) (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \int_{-3}^{0} f(t)\,dt = -6 - \frac{9\pi}{4}$ (1)

(1) $g'(x) = 2 + f(x) \stackrel{?}{=} 2 + f(-3) = 2$ (1)

b) $g'(x) = 0$ (1) when $f(x) = -2$, $x = 2.5$ (1)

$g'(x)$: $+$ | $-$

x 2.5

(1) slope goes pos to neg, therefore abs max

c) $g'(x) = 2 + f(x)$

$g''(x) = f'(x)$ $x = 0$ (1)

$f'(x)$ changes from pos to neg

d) $\frac{-3 + {+1}}{3 + (+4)} = \frac{-2}{7}$ (1) (1) f is not differentiable at $x = -3$ and 0

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