CALCULUS BC
WORKSHEET ON SERIES

Work the following on notebook paper.

1. Which of the following is a term in the Taylor series about $x=0$ for the function $f(x)=\cos(2x)$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

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2. Find the values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n(-3)^n}$ converges.

(A) $x=2$ (B) $-1\le x<5$ (C) $-1<x\le 5$ (D) $-1<x<5$ (E) All real numbers

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3. Let $f(x)=\sum_{n=1}^{\infty}(\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$.

(A) $-\frac{1}{7}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{8}{9}$ (E) The series diverges.

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4. Find the sum of the geometric series $\frac{9}{8}-\frac{3}{4}+\frac{1}{2}-\frac{1}{3}+\ldots$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

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5. The series $x+x^3+\frac{x^5}{2!}+\frac{x^7}{3!}+\ldots+\frac{x^{2n+1}}{n!}+\ldots$ is the Maclaurin series for

(A) $x\ln(1+x^2)$ (B) $x\ln(1-x^2)$ (C) e^{x^2} (D) xe^{x^2} (E) $x^2e^{x^2}$

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6. The coefficient of x^3 in the Taylor series for e^{2x} at $x=0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

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7. The Taylor polynomial of order 3 at $x=0$ for $f(x)=\sqrt{1+x}$ is

(A) $1+\frac{x}{2}-\frac{x^2}{4}+\frac{3x^3}{8}$ (B) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}$ (C) $1-\frac{x}{2}+\frac{x^2}{8}-\frac{x^3}{16}$

(D) $1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{8}$ (E) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{3x^3}{8}$

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1. Which of the following is a term in the Taylor series about $x = 0$ for the function $f(x) = \cos(2x)$?

(A) $-\frac{1}{2}x^2$ (B) $-\frac{4}{3}x^3$ (C) $\frac{2}{3}x^4$ (D) $\frac{1}{60}x^5$ (E) $\frac{4}{45}x^6$

$1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $\quad 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} = 1 - 2x^2 + \frac{2}{3}x^4$

2. Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(-3)^n}$ converges.

$x = -1: \frac{(-3)^n}{n(-3)^n} = \frac{1}{n}$ div
$x = 5: \frac{3^n}{n(-3)^n} = \frac{(-1)^n}{n}$ conv

(A) $x = 2$ (B) $-1 \le x < 5$ (C) $-1 < x \le 5$ (D) $-1 < x < 5$ (E) All real numbers

$\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{(n+1)(-3)^{n+1}} \cdot \frac{n(-3)^n}{(x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{n}{-3(n+1)} \cdot (x-2) \right| < 1$ $\quad -1 < \frac{1}{3}(x-2) < 1$

$-3 < x-2 < 3$
$-1 < x < 5$

3. Let $f(x) = \sum_{n=1}^{\infty} (\cos x)^{3n}$. Evaluate $f\left(\frac{2\pi}{3}\right)$. $\quad \left(\cos \frac{2\pi}{3}\right)^{3n}$

(A) $-\frac{1}{7}$ (B) $-\frac{1}{9}$ (C) $\frac{1}{7}$ (D) $\frac{8}{9}$ (E) The series diverges.

$\left(\left(-\frac{1}{2}\right)^3\right)^n = \left(-\frac{1}{8}\right)^n$ $\quad \frac{a}{1-r}$ $\quad \frac{-\frac{1}{8}}{1-\left(-\frac{1}{8}\right)} = \frac{-\frac{1}{8}}{\frac{9}{8}} = -\frac{1}{8} \cdot \frac{8}{9} = -\frac{1}{9}$

4. Find the sum of the geometric series $\frac{9}{8} - \frac{3}{4} + \frac{1}{2} - \frac{1}{3} + \ldots$ $\quad r = \frac{-\frac{3}{4}}{\frac{9}{8}} = -\frac{3}{4} \cdot \frac{8}{9} = -\frac{2}{3}$

(A) $\frac{3}{5}$ (B) $\frac{5}{8}$ (C) $\frac{13}{24}$ (D) $\frac{27}{8}$ (E) $\frac{27}{40}$

$\frac{a}{1-r}$ $\quad \frac{\frac{9}{8}}{1-\left(-\frac{2}{3}\right)} = \frac{9}{8} \cdot \frac{3}{5} = \frac{27}{40}$

5. The series $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \ldots + \frac{x^{2n+1}}{n!} + \ldots$ is the Maclaurin series for $\quad x + \frac{x^2}{2!} + \frac{x^3}{3!}$

(A) $x\ln(1+x^2)$ (B) $x\ln(1-x^2)$ (C) e^{x^2} (D) xe^{x^2} (E) $x^2e^{x^2}$ $\quad x^2 + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!}$

$\frac{x^{2n+1}}{n!} = \frac{x^{2n} \cdot x}{n!} = \frac{(x^2)^n \cdot x}{n!}$ $\quad x\left(x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}\right)$

$x^3 + \frac{x^5}{2!} + \frac{x^7}{3!}$

6. The coefficient of x^3 in the Taylor series for e^{2x} at $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{8}{3}$

$2x + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!}$ $\quad \frac{8x^3}{3!} = \frac{8x^3}{6} = \frac{4}{3}x^3$

7. The Taylor polynomial of order 3 at $x = 0$ for $f(x) = \sqrt{1+x}$ is

~~(A)~~ $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ (B) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ ~~(C)~~ $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$

(D) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$ ~~(E)~~ $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

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$f(x) = \sqrt{1+x} = (1+x)^{1/2}$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2} = \frac{1}{2\sqrt{1+x}}$ $f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2} = \frac{-1}{4(\sqrt{1+x})^3}$ $f''(0) = \frac{-1}{4}$

$$1 + \frac{1x^1}{2 \cdot 1!} - \frac{1x^2}{4 \cdot 2!} + \frac{3x^3}{8 \cdot 3!}$$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\frac{3}{8(\sqrt{1+x})^5}$ $f'''(0) = \frac{3}{8}$

$$\frac{3}{8 \cdot 6} = \frac{3}{48} = \frac{1}{16}$$

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2009 BC4

$$\frac{dy}{dx} = 6x^2 - x^2 y$$

$\rightarrow y = mx + b$

$\rightarrow y_{new} = \frac{dy}{dx}(\Delta x) + y_{old}$

a)

x	y	dy/dx
-1	2	4
$-\frac{1}{2}$	4	$\frac{1}{2}$
0	$\frac{17}{4}$	

$6(-1)^2 - 1^2 \cdot 2 = 6 - 2 = 4$

$y = 4(+\frac{1}{2}) + 2 = 4$

$6(-\frac{1}{2})^2 - (-\frac{1}{2})^2 \cdot 4$

$6 \cdot \frac{1}{4} - \frac{1}{4} \cdot 4 = \frac{3}{2} - 1 = \frac{1}{2}$

$y = \frac{1}{2}(\frac{1}{2}) + 4$

$\frac{1}{4} + 4 = \frac{17}{4}$

b) $\left.\frac{d^2y}{dx^2}\right|_{(-1,2)} = -12$

* $f(-1) = 2$

Euler $f'(-1) = 4$

* $f''(-1) = -12$

$$P(x) = 2 + \frac{4(x+1)^1}{1!} - \frac{12(x+1)^2}{2!}$$

$$P(x) = 2 + 4(x+1) - 6(x+1)^2$$

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$f(x) = e^{x/2}$

a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \frac{x^n}{n!}$

$$e^{x/2} = 1 + \left(\frac{x}{2}\right) + \frac{\left(\frac{x}{2}\right)^2}{2!} + \frac{\left(\frac{x}{2}\right)^3}{3!} + \dots \frac{\left(\frac{x}{2}\right)^n}{n!}$$

$$1 + \left(\frac{x}{2}\right) + \frac{x^2}{2^2 \cdot 2!} + \frac{x^3}{2^3 \cdot 3!} + \dots \frac{x^n}{2^n \cdot n!}$$

b) $g(x) = \frac{e^{\frac{x}{2}} - 1}{x} = \frac{e^{\frac{x}{2}}}{x} - \frac{1}{x}$

$$e^{x/2} - 1 = \left(\frac{x}{2}\right) + \frac{x^2}{2^2 \cdot 2!} + \frac{x^3}{2^3 \cdot 3!}$$

$$\frac{e^{x/2} - 1}{x} = \frac{\frac{x}{2}}{x} + \frac{\frac{x^2}{2^2 \cdot 2!}}{x} + \frac{\frac{x^3}{2^3 \cdot 3!}}{x}$$

$$= \frac{1}{2} + \frac{x}{2^2 \cdot 2!} + \frac{x^2}{2^3 \cdot 3!} + \dots \frac{x^n}{2^n \cdot n! \, x}$$

$n=1$ $n=2$ $n=3$

$$= \frac{x^{n-1}}{2^n \cdot n!}$$

start at $n=1$

$$\frac{x^n}{2^{n+1} \cdot (n+1)!}$$

start $n=0$

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