

$$1) x^3 + xy - 2 = 0 \quad (1, 1)$$

$$3x^2 + \left(x \frac{dy}{dx} + y\right) = 0$$

$$3x^2 + x \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-3x^2 - y}{x} \quad \frac{dy}{dx} \Big|_{(1,1)} = \frac{-3-1}{1} = -4$$

$$2) x^3 + y^3 = 6xy$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 6x \cdot \frac{dy}{dx} + y \cdot 6$$

$$(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\frac{dy}{dx} \Big|_{(3,3)} = \frac{18 - 27}{27 - 18} = \frac{-9}{9} = -1$$

$$3) x^3 + y^3 = 8$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

$$\frac{dy}{dx} \Big|_{(0,2)} = 0$$

$$0 + y^3 = 8$$

$$y = 2$$

$$\text{tan line: } y = 2$$

$$y\text{-int: } 2$$

$$\#) x = y^2$$

$$1 = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2y} = \frac{1}{2} y^{-1}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} y^{-2} \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2y^2} \cdot \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2y^2} \cdot \frac{1}{2y} = \frac{-1}{4y^3}$$

$$\frac{d^2y}{dx^2} \Big|_{y=\frac{1}{2}} = \frac{-1}{4\left(\frac{1}{2}\right)^3} = -2$$

$$5) x^2 - xy + y^2 = 7$$

$$2x - \left(x \frac{dy}{dx} + y\right) + 2y \frac{dy}{dx} = 0$$

$$2x - x \frac{dy}{dx} - y + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (-x + 2y) = y - 2x$$

$$\frac{dy}{dx} = \frac{y - 2x}{-x + 2y}$$

$$\frac{dy}{dx} \Big|_{(1, 3)} = \frac{3 - 2}{-1 + 6} = \frac{1}{5}$$

$$\frac{dy}{dx} \Big|_{(1, -2)} = \frac{-2 - 2}{-1 - 4} = \frac{-4}{-5} = \frac{4}{5} \quad -\frac{5}{4}$$

x =:

$$1 - y + y^2 = 7$$

$$y^2 - y + 1 = 7$$

$$y^2 - y - 6 = 0$$

$$(y - 3)(y + 2) = 0$$

$$y = 3, -2$$

normal:  
-5

$$6) \quad x^3 - 6xy - ky^3 = a$$

$$3x^2 - (6x \cdot \frac{dy}{dx} + y \cdot 6) - 3ky^2 = 0$$

$$3x^2 - 6x \frac{dy}{dx} - 6y - 3ky^2 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (-6x - 3ky^2) = +6y - 3x^2$$

$$\frac{dy}{dx} = \frac{+6y - 3x^2}{-6x - 3ky^2}$$

$$-1 = \frac{+6 - 0}{0 - 3k(1)^2}$$

$$-1 = \frac{+6}{-3k}$$

$$-1 = -\frac{2}{k}$$

$$-k = 2$$

$$k = +2$$

$$x = 0$$

$$k = -2$$

$$y = 1$$

$$0 - 6(0)(1) - (+2)(1)^3 = a$$

$$0 - 0 - 2 = a$$

$$a = -2$$

$$k + a = 0$$

$$7) \quad x^2 + 4y^2 = 7 + 3xy$$

$$2x + 8y \frac{dy}{dx} = 3x \left( \frac{dy}{dx} \right) + y(3)$$

$$\frac{dy}{dx} (8y - 3x) = 3y - 2x$$

$$\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x} = 0$$

$$3y - 2x = 0$$

$$3y - 2(3) = 0$$

$$3y = 6$$

$$y = 2$$

$$8) \quad x^2 + 2x + y^4 + 4y = 5$$

$$2x + 2 + 4y^3 \frac{dy}{dx} + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (4y^3 + 4) = -2x - 2$$

$$\frac{dy}{dx} = \frac{-2x - 2}{4y^3 + 4}$$

$$4y^3 + 4 = 0$$

$$4y^3 = -4$$

$$y^3 = -1$$

$$y = -1$$

$$\begin{pmatrix} 0, -1 \\ -2, -1 \end{pmatrix}$$

$$= -2$$

$$x^2 + 2x + 1 + 4 = 5$$

$$x^2 + 2x = 0$$

$$x(x+2) = 0$$

$$x = 0, -2$$