Sum of Infinite Geometric Series

WS:

1) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

$r = \frac{2}{3}$

$\frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = \boxed{3}$

2) $\sum_{n=0}^{\infty} \left(\frac{-2}{\pi}\right)^n$

$r = \frac{-2}{\pi}$

$\frac{1}{1-\left(\frac{-2}{\pi}\right)} = \frac{1}{\frac{\pi+2}{\pi}}$

$= \boxed{\frac{\pi}{\pi+2}}$

3) $\sum_{n=0}^{\infty} \left(\frac{4}{3}\right)^n$

$r = \frac{4}{3}$ ∴ diverges

4) $\frac{1}{3^3} + \frac{1}{3^4} + \frac{1}{3^5} \ldots$

$r = \frac{1}{3}$

$\frac{\frac{1}{27}}{1-\frac{1}{3}} = \frac{1}{27} \cdot \frac{3}{2}$

$= \boxed{\frac{1}{18}}$

5) $\sum_{n=0}^{\infty} \left(\frac{e}{2}\right)^n$

$r = \frac{e}{2}$

∴ diverges

6) $\sum_{n=3}^{\infty} \frac{3^n}{11^n}$

$\sum_{n=3}^{\infty} \left(\frac{3}{11}\right)^n$

$r = \frac{3}{11}$

$\frac{\frac{27}{1331}}{1-\frac{3}{11}}$

$\frac{27}{1331} \cdot \frac{11}{8} = \boxed{\frac{27}{968}}$

7) $1 + \frac{2}{7} + \frac{2^2}{7^2} + \frac{2^3}{7^3} \cdots$

$r = \frac{2}{7}$

$\frac{1}{1 - \frac{2}{7}} = \boxed{\frac{7}{5}}$

8) $\sum_{n=2}^{\infty} e^{3-2n}$

$e^3 \cdot e^{-2n}$

$e^3 \cdot (e^{-2})^n$

$r = \frac{1}{e^2}$

$\frac{\frac{1}{e}}{1 - \frac{1}{e^2}} = \frac{1}{e} \cdot \frac{e^2}{e^2 - 1}$

$= \boxed{\frac{e}{e^2 - 1}}$

9) $\sum_{n=0}^{\infty} \frac{9 \cdot 3^n + 4^{n-2}}{5^n}$ $\quad 4^n \cdot 4^{-2}$

$\sum_{n=0}^{\infty} \left(\frac{9}{5}\right)^n + \sum_{n=0}^{\infty} \frac{1}{16}\left(\frac{4}{5}\right)^n$

↓

diverges

10) $\frac{64}{49} + \frac{8}{7} + 1 + \frac{7}{8} + \cdots$

$r = \frac{7}{8}$

$\frac{\frac{64}{49}}{1 - \frac{7}{8}} = \frac{64}{49} \cdot 8$

$= \boxed{\frac{512}{49}}$

11) $\sum_{n=0}^{\infty} \frac{7 \cdot 3^n}{5^n}$

$7 \cdot \left(\frac{3}{5}\right)^n$

$r = \frac{3}{5}$

$\frac{7}{1 - \frac{3}{5}}$ $\quad 7 \cdot \frac{5}{2}$

$= \boxed{\frac{35}{2}}$

12) $\sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n}$

$\boxed{\frac{35}{3}}$

$\sum_{n=0}^{\infty} \frac{8}{5^n} + \sum_{n=0}^{\infty} \frac{2^n}{5^n}$

$8 \cdot \left(\frac{1}{5}\right)^n$ $\quad \left(\frac{2}{5}\right)^n$

$\frac{8}{1 - \frac{1}{5}} = 8 \cdot \frac{5}{4} = 10$ $\quad \frac{1}{1 - \frac{2}{5}} = \frac{5}{3}$

$$\sum_{n=3}^{\infty} 2\left(-\frac{3}{4}\right)^n$$

$r = -\frac{3}{4}$

$$\frac{\frac{-27}{32}}{1-\left(-\frac{3}{4}\right)} = -\frac{27}{56}$$

14) $$\sum_{0}^{\infty} e^{-n}$$

$r = \frac{1}{e}$ $\left(\frac{1}{e}\right)^n$

$$\therefore \frac{1}{1-\frac{1}{e}} = \frac{1}{\frac{e-1}{e}}$$

$$= \frac{e}{e-1}$$

15) B + C

16) B + C