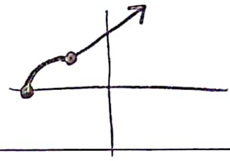
Name: Key MC_____ FR_____ Total_____ FG_____

Calculator Inactive

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$ $(25-x^2)^{1/2}$



a) Find $f'(x)$

$f'(x) = \frac{1}{2}(25-x^2)^{-1/2} \cdot -2x$

$f'(x) = \dfrac{-x}{\sqrt{25-x^2}}$

(1) $f'(x)$

b) Write an equation for the line tangent to the graph of *f* at x = -3

$f'(-3) = \dfrac{-(-3)}{\sqrt{25-(-3)^2}} = \dfrac{3}{\sqrt{16}} = \dfrac{3}{4}$

$f(-3) = \sqrt{25-(-3)^2} = \sqrt{16} = 4$

(1) $f'(-3) = \frac{3}{4}$

(1) eqn

$y - 4 = \frac{3}{4}(x+3)$

c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x+7 & \text{for } -3 < x \le 5 \end{cases}$. Is g continuous at x = -3? Use the definition of continuity to explain your answer.

(* Is g diff at x=-3?)

$\lim_{x \to -3^-} g(x) = \sqrt{25-(-3)^2} = 4$

$\lim_{x \to -3^+} g(x) = -3+7 = 4$

$\therefore \lim_{x \to -3} g(x) = 4$

$g(-3) = \sqrt{25-(-3)^2} = 4$

Yes, $\lim_{x \to -3} g(x) = g(-3) = 4$

(3) def of cont.

$\lim_{x \to -3^-} g'(x) = \frac{3}{4}$

$\lim_{x \to -3^+} g'(x) = 1$

No b/c $\lim_{x \to -3^-} g'(x) \ne \lim_{x \to -3^+} g'(x)$

d) Write the equation of the tangent line of $g(x)$ parallel to line $4x - 3y = 3$

$4x - 3y = 3$

$\dfrac{-3y}{-3} = \dfrac{-4x+3}{-3}$

$y = \frac{4}{3}x - 1$

$m = \frac{4}{3}$

$\dfrac{-x}{\sqrt{25-x^2}} = \dfrac{4}{3}$

$-3x = 4\sqrt{25-x^2}$

$\left(-\frac{3}{4}x\right)^2 = \left(\sqrt{25-x^2}\right)^2$

$\left(\frac{9}{16}x^2 = 25 - x^2\right)16$

$9x^2 = 400 - 16x^2$

$25x^2 = 400$

$x^2 = 16$

$x = \pm 4$ $x = -4$

$y = 3$

$y - 3 = \frac{4}{3}(x+4)$

(1) $m = 4/3$

(1) eqn

$g(-4) = f(-4)$

$= \sqrt{25-16} = 3$

e) Write, but do not evaluate, the limit used to find the derivative of *f*.

$\lim_{h \to 0} \dfrac{\sqrt{25-(x+h)^2} - \sqrt{25-x^2}}{h}$

(1) limit

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Calculator Active Practice question

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32^{\circ}F$, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le x \le 60$.

a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

$W'(20) = -.286$ or $-.285$

When $v = 20$ mph, the wind chill is decreasing at a rate of .286° F/mph

① value
① explain

b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

AROC: $\dfrac{W(60) - W(5)}{60 - 5} = -.254$ or $-.253$ (A)

$W'(v) = A$

$V = 23.011$ mph

① $W'(V)$=AROC ① AROC
① value of V

c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32 degrees F. At time t =0, the wind velocity is v = 20 mph. If the wind velocity ~~measures~~ increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at t = 3 hours? Indicate units of measure.

$\left.\dfrac{dW}{dt}\right|_{t=3} =$

$\left.\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right|_{t=3}$

$W'(35) \cdot 5$

$-.892$ °F/hr

$t=0$ $v=20$
$t=1$ $v=25$
$t=2$ $v=30$
$t=3$ $v=35$

① $\dfrac{dv}{dt} = 5$

① $v(3) = 35$

① ans ______/9

① units a,c

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