ANS Fundamental Theorem of Calculus WS II

1) $2\pi-\frac{5}{2}$ 2a) $2\pi-\frac{1}{2}$ 2b) -1 2c) $y-\left(2\pi-\frac{1}{2}\right)=-(x-3)$

2d) $[-4,-2]\cup[2,4]$ 2e) $[-4,0)\cup(3,5]$

3) $\frac{7}{3}$ 4) $\frac{13}{3}$ 5) $\frac{8}{3}$ 6) $\sqrt{1+x^2}$

7) $-\sqrt{1-x^2}$ 8) $2\cos\left[(2x)^2\right]$ 9) $\frac{2x}{1+\sqrt{1-x^2}}$ 10) $\csc x$

11) 168 m 12) particle traveled 168 m in 8 seconds

13) $\int_0^{12} 40(1.002)^t\,dt$ 14) $\int_0^{10} -7e^{-0.1t}\,dt$ 15) differences in masses of stars

16) 18 17) $\frac{5}{9}$ 18) $-2+\frac{1}{e}$ 19) 52 20) $-\frac{63}{4}$ 21) 1

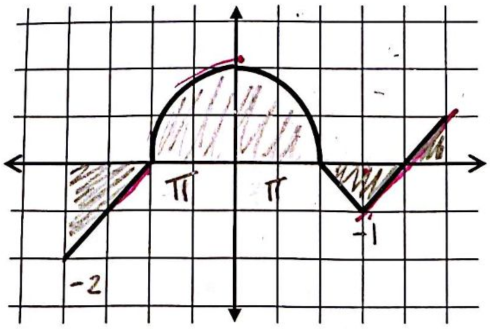
22) $\ln 3$ 23) e^2-1 24) $\frac{1}{2}e^2+e-\frac{1}{2}$

[redacted]

Sammit

AP Calculus AB
Fundamental Theorem of Calculus WS II

I. The graph of a function $h(x)$, consisting of semicircles and straight lines, is given below. Use the graph to answer the questions below. SHOW ALL WORK!



1) Evaluate $\int_{-4}^{5} h(x)\,dx$ $\quad 2\pi + \frac{1}{2} - 2 - 1$

$= 2\pi - \frac{5}{2}$

2) If $H(x) = \int_{-2}^{x} h(x)\,dx$,

$2\pi - \frac{1}{2}$ ✓

(a) $H(3) =$ $\quad (3, 2\pi - \frac{1}{2})$

(b) $h(3) =$ -1 ✓

(c) Find the equation of the tangent line of H at $x = 3$.

$H(3) = 2\pi - \frac{1}{2}$ $\quad$ slope @ $x = 3 = -1$

$y - (2\pi - \frac{1}{2}) = -1(x - 3)$

(d) Where is H decreasing?

$h(x)$ neg.

$(-4, -2) \cup (2, 4)$

(e) Where is H concave up? slope on graph is pos

$h'(x)$ pos.

$(-4, 0) \cup (3, 5)$

II. Evaluate

3) $\int_0^1 (x^2 - 2x + 3)\,dx$

$\frac{x^3}{3} - x^2 + 3x \Big|_0^1$

$\left(\frac{1}{3} - 1 + 3\right) = \frac{7}{3}$

$-\frac{3}{3} + \frac{9}{3}$

4) $\int_0^2 \sqrt{4x+1}\,dx$

$(4x+1)^{1/2}$

$\frac{(4x+1)^{3/2}}{\frac{3}{2}}$

$\frac{1}{4} \cdot \frac{2}{3}(4x+1)^{3/2}$

$\frac{1}{6}(4x+1)^{3/2} \Big|_0^2$

$\cancel{\frac{1}{4}(4x+1)^{1/2} \cdot 4}$

$\frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$

5) $\int_{-1}^{1} (x+1)^2\,dx$

$\int_{-1}^{1} x^2 + 2x + 1\,dx$

$\frac{x^3}{3} + x^2 + x \Big|_{-1}^{1}$

$\left(\frac{1}{3} + 1 + 1\right) -$

$\left(-\frac{1}{3} + 1 - 1\right)$

$\frac{7}{3} + \frac{1}{3} = \frac{8}{3}$

III. Find $F'(x)$.

6) $F(x) = \int_0^x \sqrt{1+t^2}\,dt$

$\sqrt{1+x^2}$

7) $F(x) = \int_x^1 \sqrt{1-t^2}\,dt$

$-\sqrt{1-x^2}$

8) $F(x) = \int_1^{2x} \cos(t^2)\,dt$

$\cos((2x)^2) \cdot 2$

$2\cos(4x^2)$

9) $F(x) = \int_1^{x^2} \frac{1}{1+\sqrt{1-t}}\,dt$

$\frac{1}{1+\sqrt{1-x^2}} \cdot 2x$

$\frac{2x}{1+\sqrt{1-x^2}}$

10) $F(x) = \int_{\cos x}^{0} \frac{1}{1-t^2}\,dt$

$-\frac{1}{1-(\cos x)^2} \cdot -\sin x$

$\cos^2 x$

$\csc x$

$\frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} =$

Sammit

A particle travels along a straight line with velocity given by $v(t) = 4t + 5$ m/s for time 0 to 8 seconds.

Evaluate $\int_0^8 v(t)\,dt$. $= \int_0^8 (4t+5)\,dt$

$= 2t^2 + 5t \Big|_0^8 = (128+40) - 0$

168 m

12) What does the answer to #11 mean in terms of the particle?

traveled 168 m in 8 sec

V. Write an expression for the following. DO NOT SOLVE.

13) A news broadcast in early 1993 said the average American's annual income is changing at a rate given in dollars per month by $r(t) = 40(1.002)^t$ where t is in months from January 1, 1993. If this trend continues, what change in income can the average American expect in 1993? $\int_0^{12} 40(1.002)^t$

14) A cup of coffee at 90°C is put into a 20°C room when $t = 0$. If the coffee's temperature is changing at a rate given in °C per minute by $r(t) = -7e^{-0.1t}$, t in minutes, estimate the coffee's temperature when $t = 10$. $\int_0^{10} -7e^{-0.1t}\,dt$

15) In an imaginary galaxy, a star is growing at a rate given by $r(t) = (\tan t)^{t+1}$ unit mass per century. A second star grows at a rate given by $p(t) = 2^t$ unit mass per century. Give a physical interpretation for the definite integral $\int_0^1 (2^t - (\tan t)^{t+1})\,dt$.

difference in masses of stars

VI. Evaluate each definite integral using the Fundamental Theorem of Calculus.

16) $\int_0^2 (6x^2 - 4x + 5)\,dx$ 18

17) $\int_0^1 x^{\frac{4}{5}}\,dx$ $\frac{5}{9}$

18) $\int_{-1}^0 (2x - e^x)\,dx$ $-2 + \frac{1}{e}$

19) $\int_{-2}^2 (3u+1)^2\,du$ $9u^2 + 6u + 1$ 52

20) $\int_{-2}^{-1} \left(4y^3 + \frac{2}{y^3}\right) dy$ $4y^3 + 2y^{-3}$ $-\frac{63}{4}$

21) $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$ 1

22) $\int_1^9 \frac{1}{2x}\,dx$ $\ln 3$

23) $\int_{-1}^1 e^{u+1}\,du$ $e^2 - 1$

24) $\int_1^e \frac{x^2 + x + 1}{x}\,dx$ $\frac{e^2}{2} + e - \frac{1}{2}$

16) $\frac{6x^3}{3} - \frac{4x^2}{2} + 5x \Big|_0^2$

$2x^3 - 2x^2 + 5x \Big|_0^2$

$16 - 8 + 10$

17) $\frac{5}{9} x^{9/5} \Big|_0^1$

$\frac{5}{9}$

18) $\frac{2x^2}{2} - e^x$

$= x^2 - e^x \Big|_{-1}^0$

$(0 - 1) - (1 - \frac{1}{e})$

$-1 - 1 + \frac{1}{e}$

19) $\frac{9u^3}{3} + \frac{6u^2}{2} + u \Big|_{-2}^2$

$3u^3 + 3u^2 + u$

$(24 + 12 + 2) - (-24 + 12 - 2)$

$38 - (-14) = 52$

20) $\frac{4y^4}{4} + \frac{2y^{-2}}{-2} \Big|_{-2}^{-1}$

$y^4 + -\frac{1}{y^2} \Big|_{-2}^{-1}$

$(1 + -1) - (16 + -\frac{1}{4})$

$0 - \left(\frac{64}{4} - \frac{1}{4}\right) = -\frac{63}{4}$

21) $\tan x \Big|_0^{\pi/4}$

$1 - 0 = 1$