CALCULUS
WORKSHEET 1 ON FUNDAMENTAL THEOREM OF CALCULUS

Work problems 1 - 2 by both methods. Do not use your calculator.
1. \( y' = 2 + \frac{1}{x^2} \) and \( y(1) = 6 \). Find \( y(3) \).
2. \( f'(x) = \cos(2x) \) and \( f(0) = 3 \). Find \( f\left(\frac{\pi}{4}\right) \).

Work problems 3 – 7 using the Fundamental Theorem of Calculus and your calculator.
3. \( f'(x) = \cos(x^3) \) and \( f(0) = 2 \). Find \( f(1) \).
4. \( f'(x) = e^{-x^2} \) and \( f(5) = 1 \). Find \( f(2) \).
5. A particle moving along the \( x \)-axis has position \( x(t) \) at time \( t \) with the velocity of the particle \( v(t) = 5 \sin(t^2) \). At time \( t = 6 \), the particle’s position is \((4, 0)\). Find the position of the particle when \( t = 7 \).
6. Let \( F(t) \) represent a bacteria population which is 4 million at time \( t = 0 \). After \( t \) hours, the population is growing at an instantaneous rate of \( 2^t \) million bacteria per hour. Find the total increase in the bacteria population during the first three hours, and find the population at \( t = 3 \) hours.
7. A particle moves along a line so that at any time \( t \geq 0 \) its velocity is given by \( v(t) = \frac{t}{1+t^2} \). At time \( t = 0 \), the position of the particle is \( s(0) = 5 \). Determine the position of the particle at \( t = 3 \).

Use the Fundamental Theorem of Calculus and the given graph.
8. The graph of \( f' \) is shown on the right.
\[ \int_1^4 f'(x) dx = 6.2 \] and \( f(1) = 3 \). Find \( f(4) \).

9. The graph of \( f' \) is the semicircle shown on the right.
Find \( f(-4) \) given that \( f(4) = 7 \).

10. The graph of \( f' \), consisting of two line segments and a semicircle, is shown on the right. Given that \( f(-2) = 5 \), find:
(a) \( f(1) \) \hspace{1cm} (b) \( f(4) \) \hspace{1cm} (c) \( f(8) \)

11. Region A has an area of 1.5, and \( \int_{0}^{6} f(x)\,dx = 3.5 \). Find:

(a) \( \int_{2}^{6} f(x)\,dx \)

(b) \( \int_{0}^{6} |f(x)|\,dx \)

12. The graph on the right shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower has 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.

13. A cup of coffee at 90° C is put into a 20° C room when \( t = 0 \). The coffee’s temperature is dropping at a rate of \( r(t) = 7e^{-0.3t} \)°C per minute, with \( t \) in minutes. Estimate the coffee’s temperature when \( t = 10 \).

14. Given the figure on the right and the fact that \( F(2) = 3 \).

(a) Find \( F(0), F(6), \) and \( F(8) \).

(b) Determine the intervals where the graph of \( F \) is increasing and decreasing. Justify your answer.

(c) Determine the intervals where the graph of \( F \) is concave up and concave down. Justify your answer.

(d) Use your results to sketch the graph of \( F(x) \). Label the values of at least four points.