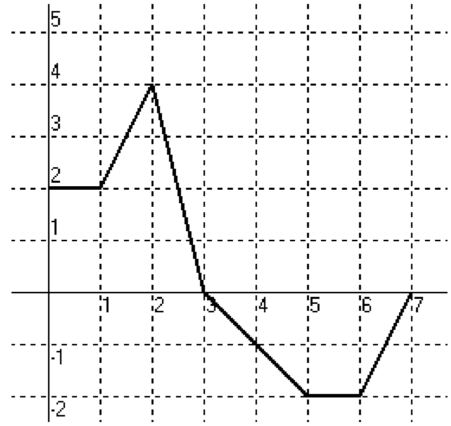
AP Calculus BC
Section 5.3 – FTC Free Response Questions



1. (Stewart – no calculator) Let $g(x)=\int_0^x f(t)dt$, where *f* is the function whose graph is shown to the right.

   a. Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

   b. On what intervals is *g* increasing?

   c. Where does g have a maximum value?

   d. Evaluate $g'(2)$

   e. Find any points of inflection. Justify your answers.

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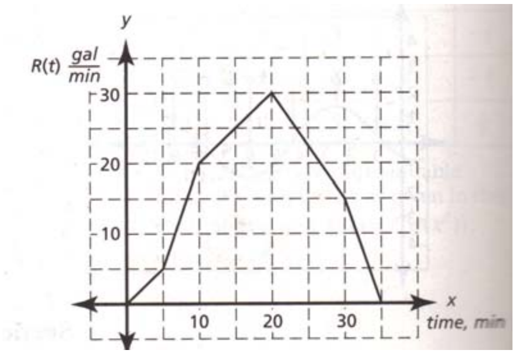
2. (Lucia – calculator) Water is draining out of a tank at a variable rate as given by the chart and graph below.

t (min)	$R(t)$ (gallons/min)
0	0
5	5
10	20
20	30
30	15
35	0



a. Approximate the volume of water that has leaked from the tank from 0 to 35 minutes using a Riemann sum with a right-hand end point for the five unequal intervals indicated by the chart.

b. Interpret the meaning of $\frac{1}{20}\int_{10}^{30} R(t)dt$ and find its value with the appropriate units using the graph.

c. Calculate $R'(25)$ with appropriate units. Justify your answer.

d. If the rate of the leak is modeled by $Q(x) = 16.78\sin(0.15x - 1.25) + 14.6$, at what time is the water leaking the fastest?

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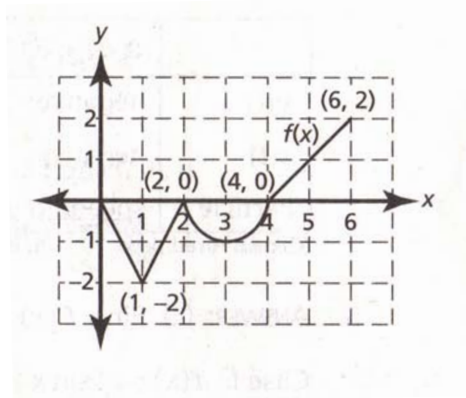

3. (Lucia – no calculator) Let f by a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle.

   Let $g(x) = 3 + \int_2^x f(t)dt$.

   a. Find $g(1)$, $g'(1)$, and $g''(1)$.

   b. What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?

   c. What is the average value of $g'(x)$ on the same interval as part b)?

   d. Identify the x – coordinate(s) of any relative extrema. Justify your answers.

   e. Identify the x – coordinate(s) of any points of inflection. Justify your answers.

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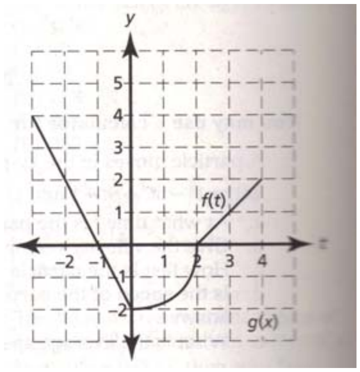
4. (Lucia – no calculator) The graph of $f(t)$, a continuous function defined on the interval $-3 \le t \le 4$, consists of two line segments and a quarter circle, as show in the figure. Let $g(x) = \int_{-3}^{x} f(t)dt$.



   a. Evaluate $g(0)$ and $g(4)$.

   b. Find the x – coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answers.

   c. Does $\lim_{x \to 2} g''(x)$ exist? Give a reason for your answer.

   d. Find the x – coordinates of all inflection points of $g(x)$. Justify your answer.

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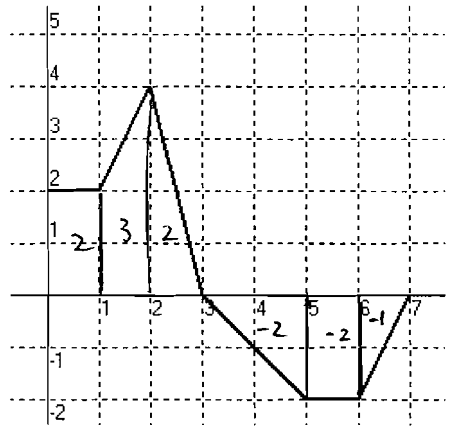
1. (Stewart – no calculator) Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown to the right.



   a. Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

   $g(0) = \int_0^0 f(t)dt = 0$ $\quad g(3) = \int_0^3 f(t)dt = 7$

   $g(1) = \int_0^1 f(t)dt = 2$ $\quad g(6) = \int_0^6 f(t)dt = 7 - 4 = 3$

   b. On what intervals is g increasing?

   g IS INCREASING WHEN $g' = f(x)$ IS POSITIVE.

   $f(x) > 0$ on $[0, 3]$

   c. Where does g have a maximum value?

   g HAS A MAX WHEN $g'(x) = f(x) = 0$ AND $g'(x) = f(x)$ CHANGES SIGN FROM POSITIVE TO NEGATIVE. THIS OCCURS AT $x = 3$

   d. Evaluate $g'(2)$

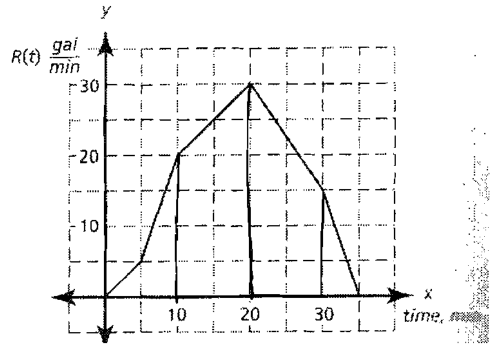
   $g'(2) = f(2) = 4$

   e. Find any points of inflection. Justify your answers.

   $g''(x) = f'(x)$ MUST CHANGE SIGN. THIS OCCURS AT $x = 2$.

2. (Lucia – calculator) Water is draining out of a tank at a variable rate as given by the chart and graph below.

t (min)	$R(t)$ (gallons/min)
0	0
5	5
10	20
20	30
30	15
35	0



a. Approximate the volume of water that has leaked from the tank from 0 to 35 minutes using a Riemann sum with a right-hand end point for the five unequal intervals indicated by the chart.

$5(5)+5(20)+10(30)+10(15)+5(0) = 575$ GALLONS.

b. Interpret the meaning of $\frac{1}{20}\int_{10}^{30} R(t)dt$ and find its value with the appropriate units using the graph.

$\frac{1}{20}\int_{10}^{30} R(t)dt = \frac{1}{20}(600-50-75) = 23.75$ GALLONS/min → AVERAGE RATE H_2O LEAKS FROM 10 TO 30 MINUTES.

c. Calculate $R'(25)$ with appropriate units. Justify your answer.

$R'(25) = \frac{15-30}{10} = -1.5$ GAL/min^2

d. If the rate of the leak is modeled by $Q(x) = 16.78\sin(0.15x - 1.25) + 14.6$, at what time is the water leaking the fastest?

$Q'(x) = (.15)(16.78)\cos(.15x - 1.25) = 0$ AT $x = 18.805$

$Q''(x) = -(.15)^2(16.78)\sin(.15x - 1.25)$

$Q''(18.805) = -.3776 < 0$ ∴ MAX AT $x = 18.805$

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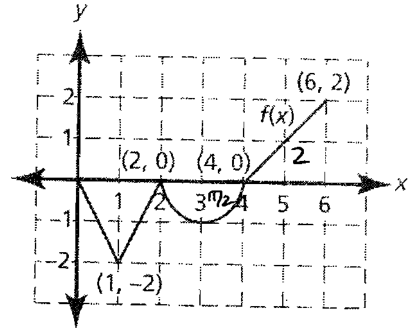
3. (Lucia – no calculator) Let f by a function defined in the closed interval $0 \le x \le 6$. The graph of f consists of three line segments and a semicircle. Let $g(x) = 3 + \int_2^x f(t)dt$.



a. Find $g(1)$, $g'(1)$, and $g''(1)$.

$g(1) = 3 + \int_2^1 f(t)dt = 3+1 = 4$

$g'(1) = f(1) = -2$

$g''(1) = f'(1) \to$ DNE

$\int_2^1 f(t)dt = -\int_1^2 f(t)dt$

$-(-1)$

These are the same

b. What is the average rate of change of $g(x)$ in the interval $2 \le x \le 6$?

$$\frac{g(6)-g(2)}{6-2} = \frac{(\cancel{3}+\int_2^6 f(t)dt) - (\cancel{3+\int_2^2 f(t)dt})}{4}$$

$$= \frac{2-\pi/2}{4} = \frac{4-\pi}{8}$$

c. What is the average value of $g'(x)$ on the same interval as part b)?

$$g'_{AVE} = \frac{1}{4}\int_2^6 f(t)dt$$

$$= \frac{1}{4}\left(2-\pi/2\right) = \frac{4-\pi}{8}$$

d. Identify the x – coordinate(s) of any relative extrema. Justify your answers.

$g'(x) = f(x) = 0$ AT $x = 2, 4$ SINCE $g'(x)$ GOES FROM $-$ TO $+$ AT $x=4$, $x=4$ IS A RELATIVE MIN.

SIGN TEST FOR $g'(x)$:

$-\,-$, $-\,-$, $+\,+$

0 2 4 6

e. Identify the x – coordinate(s) of any points of inflection. Justify your answers.

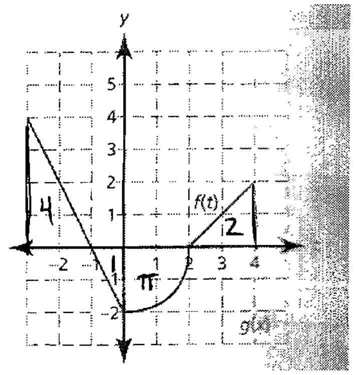
NEED $g''(x)$ TO CHANGE SIGN $(g''(x) = f'(x))$

$-$, $+$, $-$, $+$, $+$

0 1 2 3 4 6

POINTS OF INFLECTION AT $x = 1, 2, 3$

4. (Lucia – no calculator) The graph of $f(t)$, a continuous function defined on the interval $-3 \le t \le 4$, consists of two line segments and a quarter circle, as show in the figure. Let $g(x) = \int_{-3}^{x} f(t)\,dt$.



   a. Evaluate $g(0)$ and $g(4)$.

   $g(0) = \int_{-3}^{0} f(t)dt = 3$

   $g(4) = \int_{-3}^{4} f(t)dt = 5 - \pi$

   b. Find the x – coordinate of the absolute maximum and absolute minimum of $g(x)$. Justify your answers.

   $g'(x) = f(x) = 0$ AT $x = -1, 2$

   $+$ $-$ $+$ on -3, -1, 2, 4

   SINCE g' CHANGES FROM $+$ TO $-$, MAX AT $x = -1$.

   SINCE g' CHANGES FROM $-$ TO $+$, MIN AT $x = 2$

   c. Does $\lim_{x \to 2} g''(x)$ exist? Give a reason for your answer.

   $g'' = f'$

   $\lim_{x \to 2^+} f' = 1$

   $\lim_{x \to 2^-} f' = \infty$

   SINCE THE LEFT AND RIGHT HANDED LIMITS ARE DIFFERENT, $\lim_{x \to 2} g''(x)$ DNE.

   d. Find the x – coordinates of all inflection points of $g(x)$. Justify your answer.

   NEED $g''(x)$ TO CHANGE SIGN

   $g''(x) = f'(x)$

   $-$ $+$ $+$ on -3, 0, 2

   P.O.I. AT $x = 0$