Final Exam Review Day 1

Date________________ Period____

For each problem, find all points of absolute minima and maxima on the given interval.

1) $y = -\frac{x^2}{2} - 2x - 2$; $[-3, -1]$

2) $y = -x^2 + 8x - 12$; $[1, 6]$

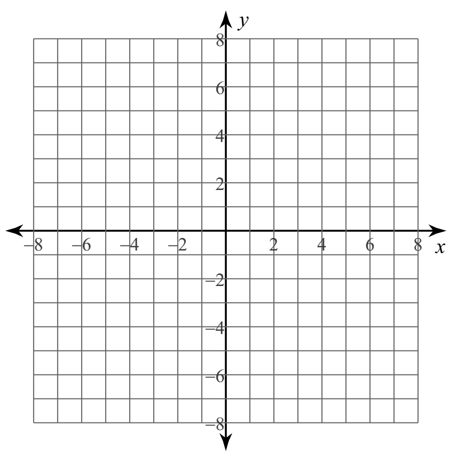
For each problem, find the average rate of change of the function over the given interval.

3) $y = -x^2 - 1$; $[1, 2]$

4) $y = -x^2 - 1$; $[0, 2]$

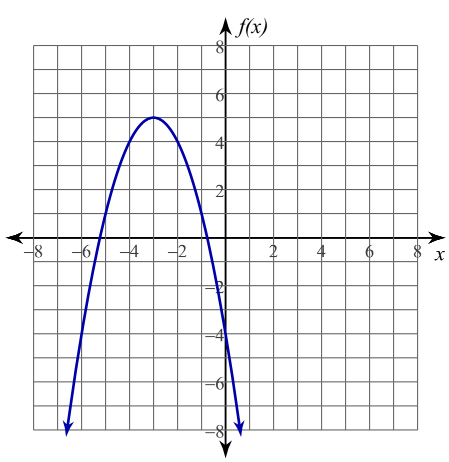
For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

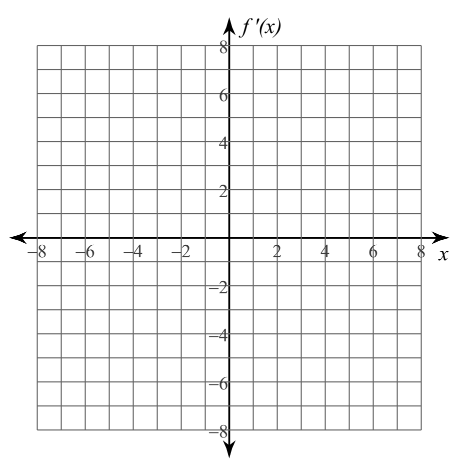
5) $y = -\frac{x^2}{2} + 3x + \frac{3}{2}$



Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

6)





For each problem, find the open intervals where the function is concave up and concave down.

7) $y = -\frac{x^2}{2} + 2x + 3$

8) $y = 2x^2 + 8x + 8$

For each problem, find the open intervals where the function is increasing and decreasing.

9) $y = \frac{x^2}{2} - 2x + 3$

10) $y = -\frac{x^2}{2} + 2x + 4$

For each problem, find the values of c that satisfy the Mean Value Theorem.

11) $y = \frac{x^2}{2} - 4x + 8;\ \ [3, 7]$

12) $y = -\frac{x^2}{2} - 4x - 4;\ \ [-5, -3]$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.

13) $s(t) = t^3 - 24t^2 + 144t$

14) $s(t) = t^3 - t^2 - 56t$

Solve each optimization problem.

15) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

Solve each related rate problem.

16) A crowd gathers around a movie star, forming a circle. The radius of the crowd increases at a rate of 7 ft/sec. How fast is the area taken up by the crowd increasing when the radius is 14 ft?

17) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 4 m/min. How fast is the area of the spill increasing when the radius is 10 m?

For each problem, find all points of relative minima and maxima.

18) $y = -x^2 - 4x - 3$

19) $y = -2x^2 - 8x - 7$

For each problem, find the values of *c* that satisfy Rolle's Theorem.

20) $y = -2x^2 - 8x - 7; \quad [-3, -1]$

21) $y = x^2 - 2x - 5; \quad [0, 2]$

For each problem, find the derivative of the function at the given value.

22) $y = 2x^2 - 16x + 33$ at $x = 3$

23) $y = -x^2 - 4x - 1$ at $x = -3$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

24) $1 = 4x^3 - 5y^2$

25) $2x^2 - 4y^2 = 3$

Differentiate each function with respect to *x*.

26) $y = \sin^{-1} -x^5$

27) $y = \sin^{-1} -x^4$

28) $y = \ln 2x^3$

29) $y = \ln 4x^3$

30) $y = \log_5 x^2$

31) $y = 4^{3x^5}$

32) $y = -2x^2$

33) $y = 5x^3$

34) $y = (5x^5 + 1) \cdot -x^5$

35) $y = -x^4(-4x^5 + 2)$

36) $y = \frac{x^2}{2x^4 - 2}$

37) $y = \frac{2x^5}{x^3 - 5}$

38) $y = \cos 4x^4$

39) $y = \cos 4x^3$

For each problem, find the indicated derivative with respect to *x*.

40) $y = 5x^4$ Find $\frac{d^2y}{dx^2}$

41) $y = -x^5$ Find $\frac{d^2y}{dx^2}$

Evaluate each limit.

42) $\lim_{x \to -3^+} -\frac{1}{x + 3}$

43) $\lim_{x \to -2^+} -\frac{x}{x + 2}$

44) $\lim_{x \to -\infty} (-x^3 + 4x^2 - 6)$

45) $\lim_{x \to -\infty} (2x^2 - 16x + 31)$

46) $\lim_{x \to -1} \frac{x + 1}{x^2 - x - 2}$

47) $\lim_{x \to -3} -\frac{x^2 - x - 12}{x + 3}$

48) $\lim_{x \to 3} (-2x + 5)$

49) $\lim_{x \to 3} 2x$

Answers to Final Exam Review Day 1 (ID: 1)

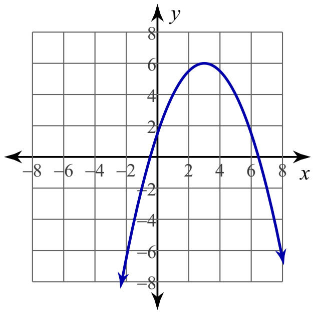
1) Absolute minima: $\left(-3, -\frac{1}{2}\right), \left(-1, -\frac{1}{2}\right)$
Absolute maximum: $(-2, 0)$

2) Absolute minimum: $(1, -5)$
Absolute maximum: $(4, 4)$

3) -3

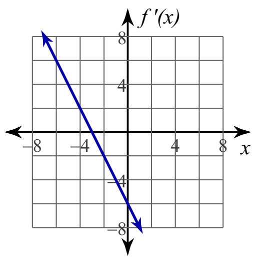
4) -2

5)



x-intercepts at $x = 3 - 2\sqrt{3}, 3 + 2\sqrt{3}$ y-intercept at $y = \frac{3}{2}$
Critical point at: $x = 3$
Increasing: $(-\infty, 3)$ Decreasing: $(3, \infty)$
No inflection points exist.
Concave up: No intervals exist. Concave down: $(-\infty, \infty)$
No relative minima. Relative maximum: $(3, 6)$

6)



7) Concave up: No intervals exist. Concave down: $(-\infty, \infty)$

8) Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.

9) Increasing: $(2, \infty)$ Decreasing: $(-\infty, 2)$

10) Increasing: $(-\infty, 2)$ Decreasing: $(2, \infty)$

11) $\{5\}$

12) $\{-4\}$

13) $v(t) = 3t^2 - 48t + 144,\ a(t) = 6t - 48$

14) $v(t) = 3t^2 - 2t - 56,\ a(t) = 6t - 2$

15) 2 in

16) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 7$ Find: $\left.\frac{dA}{dt}\right|_{r = 14}$

$$\left.\frac{dA}{dt}\right|_{r = 14} = 2\pi r \cdot \frac{dr}{dt} = 196\pi \text{ ft}^2/\text{sec}$$

17) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{r = 10}$

$$\left.\frac{dA}{dt}\right|_{r = 10} = 2\pi r \cdot \frac{dr}{dt} = 80\pi \text{ m}^2/\text{min}$$

18) No relative minima.
Relative maximum: $(-2, 1)$

19) No relative minima.
Relative maximum: $(-2, 1)$

20) $\{-2\}$

21) $\{1\}$

22) $\left.\frac{dy}{dx}\right|_{x = 3} = -4$

23) $\left.\frac{dy}{dx}\right|_{x = -3} = 2$

24) $\frac{dy}{dx} = \frac{6x^2}{5y}$

25) $\frac{dy}{dx} = \frac{x}{2y}$

26) $\frac{dy}{dx} = \frac{1}{\sqrt{1 - (-x^5)^2}} \cdot -5x^4$
$= -\frac{5x^4}{\sqrt{1 - x^{10}}}$

27) $\frac{dy}{dx} = \frac{1}{\sqrt{1 - (-x^4)^2}} \cdot -4x^3$
$= -\frac{4x^3}{\sqrt{1 - x^8}}$

28) $\frac{dy}{dx} = \frac{1}{2x^3} \cdot 6x^2$
$= \frac{3}{x}$

29) $\frac{dy}{dx} = \frac{1}{4x^3} \cdot 12x^2$
$= \frac{3}{x}$

30) $\frac{dy}{dx} = \frac{1}{x^2 \ln 5} \cdot 2x$
$= \frac{2}{x \ln 5}$

31) $\frac{dy}{dx} = 4^{3x^5} \ln 4 \cdot 15x^4$

32) $\frac{dy}{dx} = -4x$

33) $\frac{dy}{dx} = 15x^2$

34) $\frac{dy}{dx} = (5x^5 + 1) \cdot -5x^4 - x^5 \cdot 25x^4$
$= -50x^9 - 5x^4$

35) $\frac{dy}{dx} = -x^4 \cdot -20x^4 + (-4x^5 + 2) \cdot -4x^3$
$= 36x^8 - 8x^3$

36) $\frac{dy}{dx} = \frac{(2x^4 - 2) \cdot 2x - x^2 \cdot 8x^3}{(2x^4 - 2)^2}$
$= \frac{-x^5 - x}{x^8 - 2x^4 + 1}$

37) $\frac{dy}{dx} = \frac{(x^3 - 5) \cdot 10x^4 - 2x^5 \cdot 3x^2}{(x^3 - 5)^2}$
$= \frac{4x^7 - 50x^4}{x^6 - 10x^3 + 25}$

38) $\frac{dy}{dx} = -\sin 4x^4 \cdot 16x^3$
$= -16x^3 \sin 4x^4$

39) $\frac{dy}{dx} = -\sin 4x^3 \cdot 12x^2$
$= -12x^2 \sin 4x^3$

40) $\frac{d^2y}{dx^2} = 60x^2$

41) $\frac{d^2y}{dx^2} = -20x^3$

42) $-\infty$

43) ∞

44) ∞

45) ∞

46) $-\frac{1}{3}$

47) 7

48) -1

49) 6

