1) Calc Active

Let $R$ be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis.

(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles. Find the volume of this solid.

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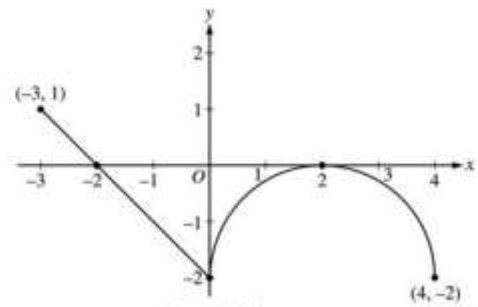
2) Calc Inactive

Let $f$ be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.

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(b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

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(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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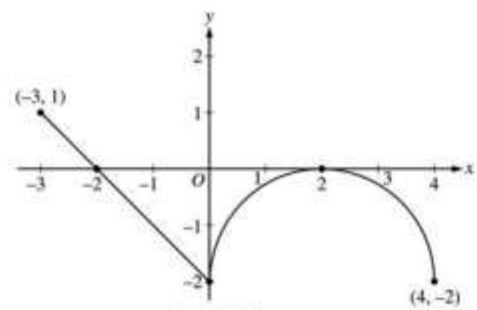

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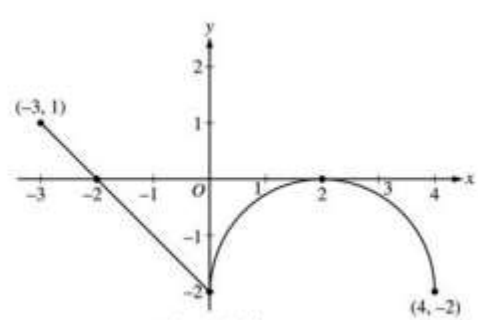

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3) Calc Active

A particle moves along the $x$-axis so that its velocity at time $t$ is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

(a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?

(b) Find all times $t$ in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

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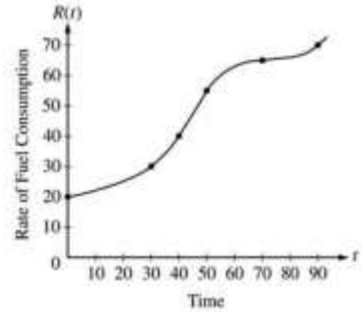
4) Calc Inactive

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function $R$ of time $t$. The graph of $R$ and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.




| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$ ? Explain your reasoning.

(d) For $0 < b \le 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

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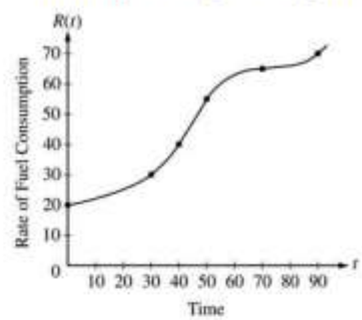

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1)

| | |
|---|---|
| $\frac{20}{1+x^2} = 2$ when $x = \pm 3$ | 1 : correct limits in an integral in (a), (b), or (c) |
| (a) Area $= \int_{-3}^{3} \left( \frac{20}{1+x^2} - 2 \right) dx = 37.961$ or $37.962$ | $2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$ |
| (b) Volume $= \pi \int_{-3}^{3} \left( \left( \frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$ | $3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$ |
| (c) Volume $= \frac{\pi}{2} \int_{-3}^{3} \left( \frac{1}{2} \left( \frac{20}{1+x^2} - 2 \right) \right)^2 dx$ $= \frac{\pi}{8} \int_{-3}^{3} \left( \frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$ | $3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$ |

2)

(a) The function $f$ is increasing on $[-3,-2]$ since $f' > 0$ for $-3 \le x < -2$.

$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

(b) $x = 0$ and $x = 2$

$f'$ changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

$2 : \begin{cases} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{cases}$

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

(d) $f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt$

$= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$

$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$

$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt$

$= -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$

$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$

4 :
- 1 : $\pm\left(\frac{1}{2} - 2\right)$ (difference of areas of triangles)
- 1 : answer for $f(-3)$ using FTC
- 1 : $\pm\left(8 - \frac{1}{2}(2)^2\pi\right)$ (area of rectangle − area of semicircle)
- 1 : answer for $f(4)$ using FTC

3)

| | |
|---|---|
| (a) $a(2) = v'(2) = 1.587$ or $1.588$<br>$v(2) = -3\sin(2) < 0$<br>Speed is decreasing since $a(2) > 0$ and $v(2) < 0$. | $2: \begin{cases} 1: & a(2) \\ 1: & \text{speed decreasing with reason} \end{cases}$ |
| (b) $v(t) = 0$ when $\frac{t^2}{2} = \pi$<br>$t = \sqrt{2\pi}$ or $2.506$ or $2.507$<br>Since $v(t) < 0$ for $0 < t < \sqrt{2\pi}$ and $v(t) > 0$ for $\sqrt{2\pi} < t < 3$, the particle changes directions at $t = \sqrt{2\pi}$. | $2: \begin{cases} 1: & t = \sqrt{2\pi} \text{ only} \\ 1: & \text{justification} \end{cases}$ |
| (c) Distance $= \int_0^3 \lvert v(t) \rvert \, dt = 4.333$ or $4.334$ | $3: \begin{cases} 1: & \text{limits} \\ 1: & \text{integrand} \\ 1: & \text{answer} \end{cases}$ |
| (d) $\int_0^{\sqrt{2\pi}} v(t)\, dt = -3.265$<br>$x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t)\, dt = -2.265$<br>Since the total distance from $t = 0$ to $t = 3$ is 4.334, the particle is still to the left of the origin at $t = 3$. Hence the greatest distance from the origin is 2.265. | $2: \begin{cases} 1: & \pm \text{ (distance particle travels while velocity is negative)} \\ 1: & \text{answer} \end{cases}$ |

4)

(a) $R'(45) \approx \dfrac{R(50) - R(40)}{50 - 40} = \dfrac{55 - 40}{10}$
$= 1.5 \text{ gal/min}^2$

$2: \begin{cases} 1: \text{a difference quotient using numbers from table and interval that contains 45} \\ 1: 1.5 \text{ gal/min}^2 \end{cases}$

(b) $R''(45) = 0$ since $R'(t)$ has a maximum at $t = 45$.

$2: \begin{cases} 1: R''(45) = 0 \\ 1: \text{reason} \end{cases}$

(c) $\int_0^{90} R(t)\,dt \approx (30)(20) + (10)(30) + (10)(40)$
$+ (20)(55) + (20)(65) = 3700$

Yes, this approximation is less because the graph of $R$ is increasing on the interval.

$2: \begin{cases} 1: \text{value of left Riemann sum} \\ 1: \text{"less" with reason} \end{cases}$

(d) $\int_0^b R(t)\,dt$ is the total amount of fuel in gallons consumed for the first $b$ minutes.

$\frac{1}{b}\int_0^b R(t)\,dt$ is the average value of the rate of fuel consumption in gallons/min during the first $b$ minutes.

$3: \begin{cases} 2: \text{meanings} \\ \quad 1: \text{meaning of } \int_0^b R(t)\,dt \\ \quad 1: \text{meaning of } \frac{1}{b}\int_0^b R(t)\,dt \\ \quad < -1 > \text{ if no reference to time } b \\ 1: \text{units in both answers} \end{cases}$