AP Practice 4 n 1: Active

1) *Example*: [1988 AP Calculus BC #43] Bacteria in a certain culture increase at rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

A) $\frac{3\ln 3}{\ln 2}$ B) $\frac{2\ln 3}{\ln 2}$ C) $\frac{\ln 3}{\ln 2}$ D) $\ln\left(\frac{27}{2}\right)$ E) $\ln\left(\frac{9}{2}\right)$

$y = Ce^{kt}$ $\quad y = e^{kt}$ $\quad y = e^{\frac{\ln 2}{3}t}$ $\quad t=0, y=1$

$1 = Ce^0$ $\quad 2 = e^{3k}$ $\quad 3 = e^{\frac{\ln 2}{3}\cdot t}$ $\quad t=3, y=2$

$C = 1$ $\quad k = \frac{\ln 2}{3}$ $\quad t=?, y=3$

$\frac{\ln 3}{\frac{\ln 2}{3}} = \ln 3 \cdot \frac{3}{\ln 2}$

2) *Example*: [AP Calculus 1993 AB #42] A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

$t=0, y=2$ $\quad t=2, y=3.5$ $\quad t=3, y=?$

A) 4.2 pounds B) 4.6 pounds C) 4.8 pounds D) 5.6 pounds E) 6.5 pounds

$y = Ce^{kt}$ $\quad y = 2e^{kt}$ $\quad y = 2e^{\frac{\ln\frac{7}{4}}{2}\cdot 3}$

$2 = Ce^0$ $\quad \frac{7}{2} = 2e^{k(2)}$ $\quad y = 2\cdot e^{\ln\left(\frac{7}{4}\right)^{3/2}}$

$C = 2$ $\quad \frac{7}{4} = e^{2k}$ $\quad k = \frac{\ln\frac{7}{4}}{2}$ $\quad y = 2\left(\frac{7}{4}\right)^{3/2} \approx 4.63$ lbs

3) *Example*: [1993 AP Calculus BC #38] During a certain epidemic, the number of people that are infected at any time increases at rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

$t=0, y=1000$ $\quad t=7, y=1200$ $\quad t=12, y=?$

A) 343 B) 1,343 C) 1,367 D) 1,400 E) 2,057

$y = Ce^{kt}$

$y = 1000e^{kt}$ $\quad \frac{\ln\frac{6}{5}}{7} = k$

$1200 = 1000e^{7k}$

$\frac{6}{5} = e^{7k}$ $\quad y = 1000e^{\left(\frac{\ln\frac{6}{5}}{7}\right)(12)} = 1000e^{\frac{12}{7}\ln\frac{6}{5}}$

$= 1000e^{\ln\left(\frac{6}{5}\right)^{12/7}} = 1000\left(\frac{6}{5}\right)^{12/7} = 1366.908$

4) *Example*: [1998 AP Calculus AB #84] Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

$t=0, y=1$ $\quad t=10, y=2$

A) 0.069 B) 0.200 C) 0.301 D) 3.322 E) 5.000

$y = Ce^{kt}$

$y = e^{kt}$

$2 = e^{10k}$

$k = \frac{\ln 2}{10} \approx 0.069$

96 AB3/BC3

The rate of consumption of cola in the United States is given by $S(t) = Ce^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.

(a) The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.

$t=0,\ S=6$
$t=5,\ S=12$

(b) Find the average rate of consumption of cola over the 10-year time period beginning January 1, 1983. Indicate units of measure.

(c) Use the trapezoidal rule with four equal subdivisions to estimate $\int_5^7 S(t)\,dt$.

(d) Using correct units, explain the meaning of $\int_5^7 S(t)\,dt$ in terms of cola consumption.

$S=6e^{\left(\frac{\ln 2}{5}\right)(t)}$

a)	$S=Ce^{kt}$ $6=Ce^{0}$ $C=6$ 1 $12=6e^{5k}$ $2=e^{5k}$ $k=\frac{\ln 2}{5} \approx 0.138$ or 0.139 1
b)	1 $\dfrac{\int_3^{13} 6e^{\left(\frac{\ln 2}{5}\right)(t)}\,dt}{13-3} = 19.680$ billion gal/year 1
c)	$\int_5^7 S(t)dt = \frac{1}{2}\left(\frac{1}{2}\right)\left[S(5)+2\cdot S(5.5)+2\cdot S(6)+2\cdot S(6.5)+S(7)\right]$ $\frac{1}{4}(12+\overset{A}{25.723}+\overset{B}{27.569}+\overset{C}{29.547}+\overset{D}{15.834}) = 27.668$ billions of gal 1 trap 1 sum
d)	total consumption, in billions of gallons, during years 1985 and 1986. 1 consumption

1 bil of gal over time period   1 units in b, c