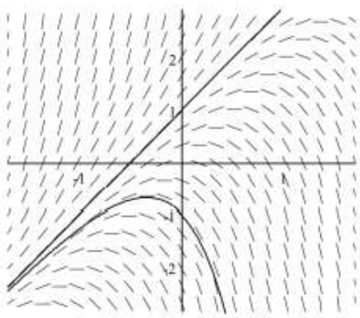
Euler’s Method HW FRQs:

9.

(a)



$2\begin{cases} 1: \text{ solution curve through } (0,1) \\ 1: \text{ solution curve through } (0,-1) \end{cases}$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) $f(0.1) \approx f(0) + f'(0)(0.1)$

$= 1 + (2 - 0)(0.1) = 1.2$

$f(0.2) \approx f(0.1) + f'(0.1)(0.1)$

$\approx 1.2 + (2.4 - 0.4)(0.1) = 1.4$

$2\begin{cases} 1: \text{ Euler's method equations or equivalent table applied to (at least) two iterations} \\ 1: \text{ Euler approximation to } f(0.2) \text{ (not eligible without first point)} \end{cases}$

(c) Substitute $y = 2x + b$ in the DE:

$2 = 2(2x + b) - 4x = 2b$, so $b = 1$

OR

Guess $b = 1$, $y = 2x + 1$

Verify: $2y - 4x = (4x + 2) - 4x = 2 = \dfrac{dy}{dx}$.

$2\begin{cases} 1: \text{ uses } \dfrac{d}{dx}(2x + b) = 2 \text{ in DE} \\ 1: \ b = 1 \end{cases}$

(d) g has local maximum at $(0,0)$.

$g'(0) = \left.\dfrac{dy}{dx}\right|_{(0,0)} = 2(0) - 4(0) = 0,$ and

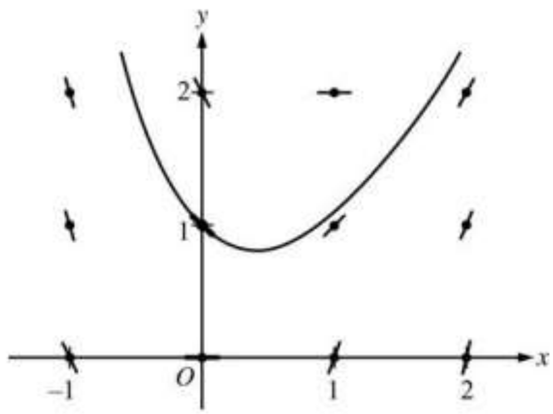
$g''(x) = \dfrac{d^2y}{dx^2} = 2\dfrac{dy}{dx} - 4,$ so

$g''(0) = 2\,g'(0) - 4 = -4 < 0.$

$3\begin{cases} 1: \ g'(0) = 0 \\ 1: \text{ shows } g''(0) = -4 \\ 1: \text{ conclusion} \end{cases}$

10.

(a)



$3: \begin{cases} 1: \text{zero slopes} \\ 1: \text{nonzero slopes} \\ 1: \text{curve through } (0, 1) \end{cases}$

(b) $\frac{dy}{dx} = 0$ when $2x = y$

The y-coordinate is $2\ln\left(\frac{3}{2}\right)$.

$2: \begin{cases} 1: \text{sets } \frac{dy}{dx} = 0 \\ 1: \text{answer} \end{cases}$

(c) $f(-0.2) \approx f(0) + f'(0)(-0.2)$
$= 1 + (-1)(-0.2) = 1.2$

$f(-0.4) \approx f(-0.2) + f'(-0.2)(-0.2)$
$\approx 1.2 + (-1.6)(-0.2) = 1.52$

$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{Euler approximation to } f(-0.4) \end{cases}$

(d) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - 2x + y$

$\frac{d^2y}{dx^2}$ is positive in quadrant II because $x < 0$ and $y > 0$.

$1.52 < f(-0.4)$ since all solution curves in quadrant II are concave up.

$2: \begin{cases} 1: \frac{d^2y}{dx^2} \\ 1: \text{answer with reason} \end{cases}$