

KEY

Derivatives, Rates of Change, and Equations of Lines

1) Find the slope of $y = (x + 1)(x + 2)$ at $x = -3$. -3

2) Find the equation of the tangent line in problem #1 at the point $(-3, 10)$.

$$y - 10 = -3(x + 3)$$

3) Find the slope of the secant line (average rate of change) over the interval $[1, 4]$ of $y = \frac{1}{\sqrt{x}}$. $-\frac{1}{6}$

4) Find the instantaneous rate of change in problem #3 at $x = 4$.

$$-\frac{1}{16}$$

5) Determine the x -value of the point in the interval $(0, 2\pi)$ where the function $y = x + \cos x$ has a horizontal tangent line.

$$\frac{\pi}{2}$$

6) Determine the equation of the vertical tangent line of $y = 2x + 3\sqrt{x}$.

$$x = 0$$

$$y - 4 = -\frac{1}{8}(x - 2)$$

7) Find the equation of the normal line (perpendicular to the tangent line) of $y = 3x^2 - 4x$ at $x = 2$.

$$1) y = x^2 + 3x + 2 \quad 2) y - 10 = -3(x + 3)$$

$$y' = 2x + 3$$

$$f'(-3) = 2(-3) + 3$$

$$f'(-3) = -3$$

$$3) y = \frac{1}{\sqrt{x}}$$

$$\frac{f(4) - f(1)}{4 - 1}$$

$$\frac{\frac{1}{2} - 1}{3} = -\frac{1}{2} \cdot \frac{1}{3} = -\frac{1}{6}$$

$$4) y = x^{-1/2}$$

$$y' = -\frac{1}{2}x^{-3/2} \quad f'(4) = -\frac{1}{2}(4)^{-3/2}$$

$$y' = -\frac{1}{2(\sqrt{x})^3} \quad f'(4) =$$

$$\frac{-1}{2 \cdot 4^{3/2}}$$

$$= \frac{-1}{2 \cdot (\sqrt{4})^3}$$

$$= -\frac{1}{16}$$

$$5) y = x + \cos x$$

$$y' = 1 - \sin x$$

$$0 = 1 - \sin x$$

$$-1 = -\sin x$$

$$1 = \sin x$$

$$x = \frac{\pi}{2}$$

$$6) y = 2x + 3x^{1/2}$$

$$y' = 2 + \frac{3}{2}x^{-1/2}$$

$$y' = 2 + \frac{3}{2\sqrt{x}}$$

$$2\sqrt{x} = 0$$

$$x = 0$$

$$7) y' = 6x - 4$$

$$f'(2) = 8 \quad (2, 4)$$

$$y - 4 = -\frac{1}{8}(x - 2)$$