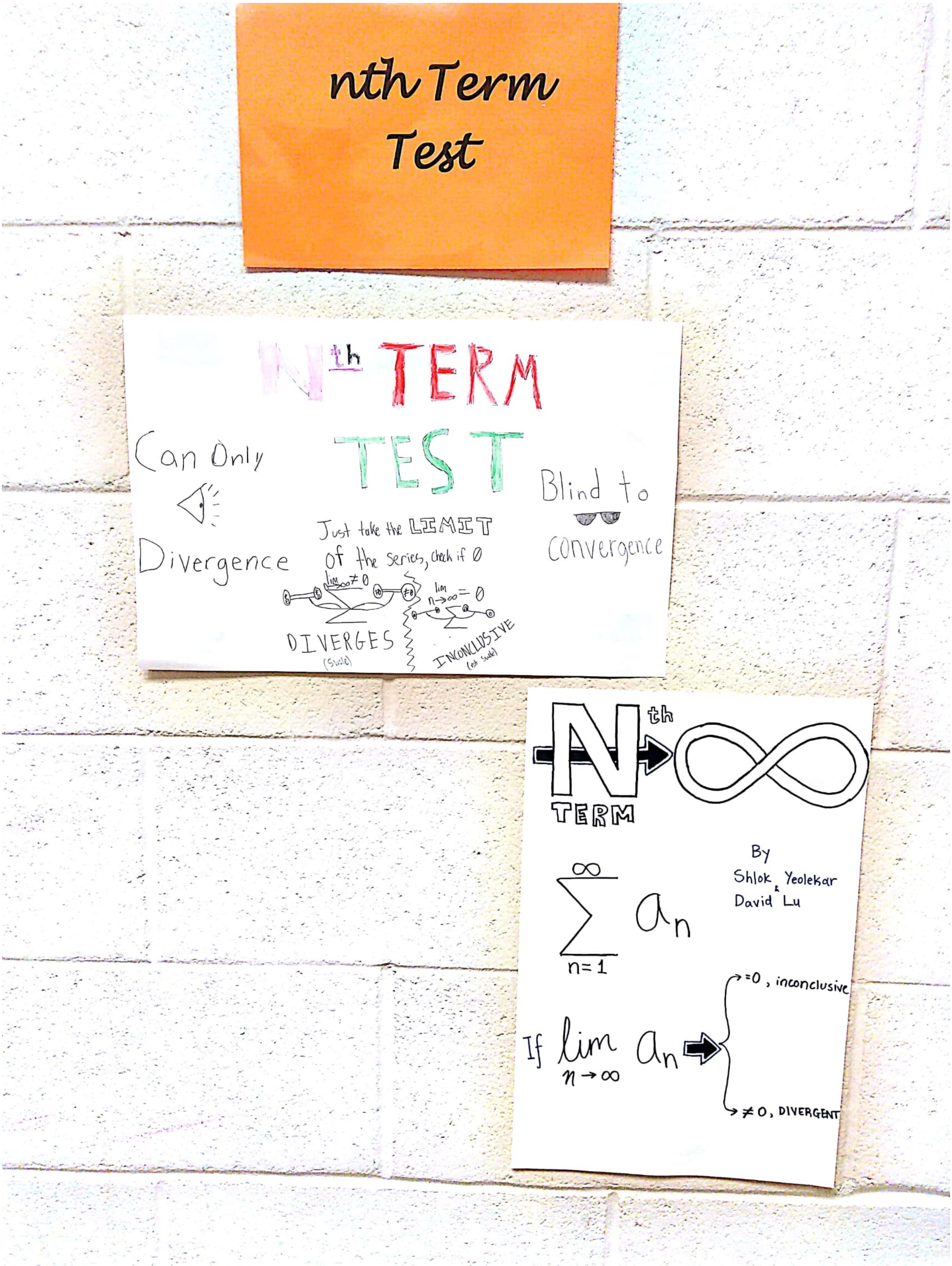

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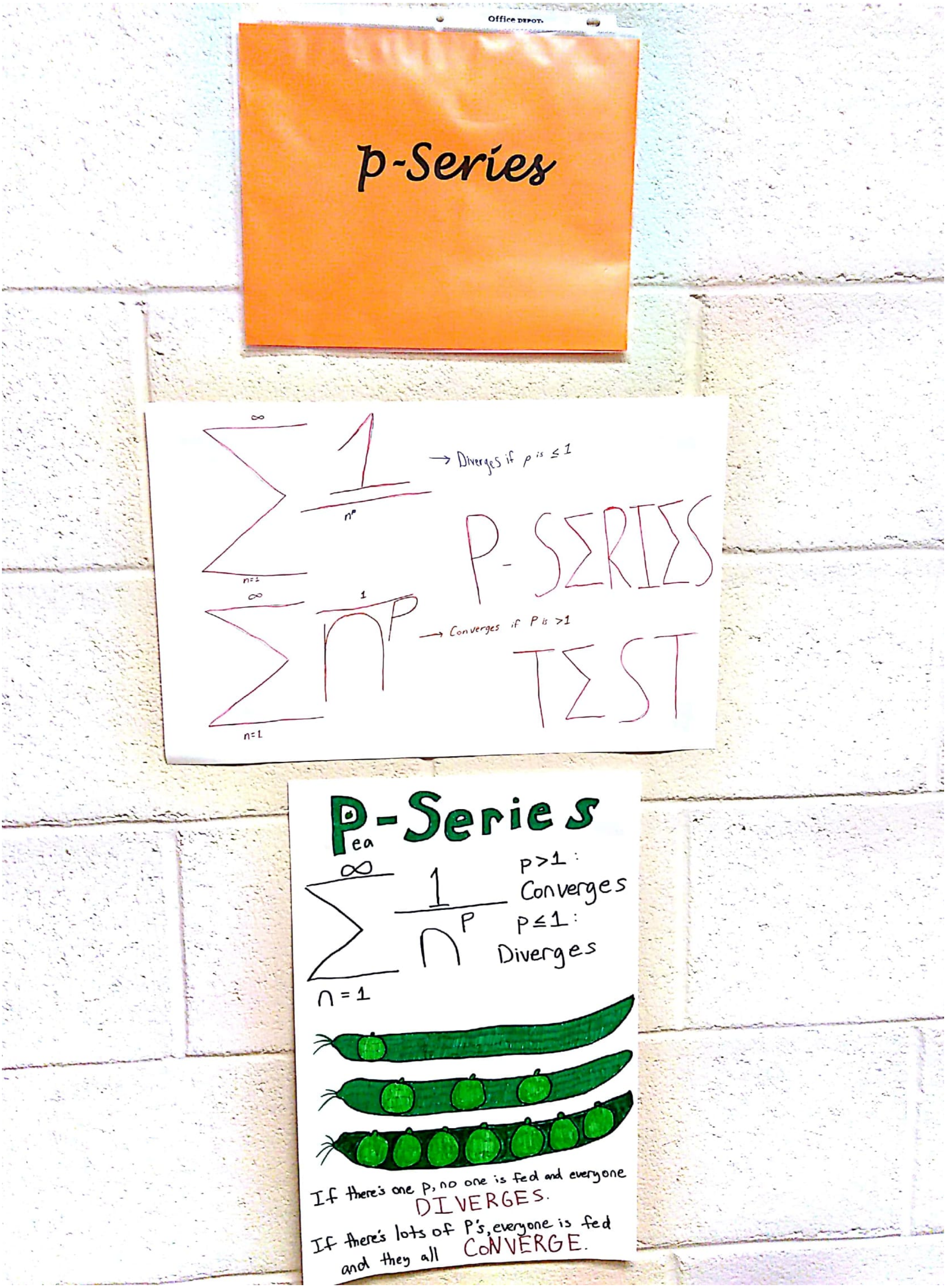
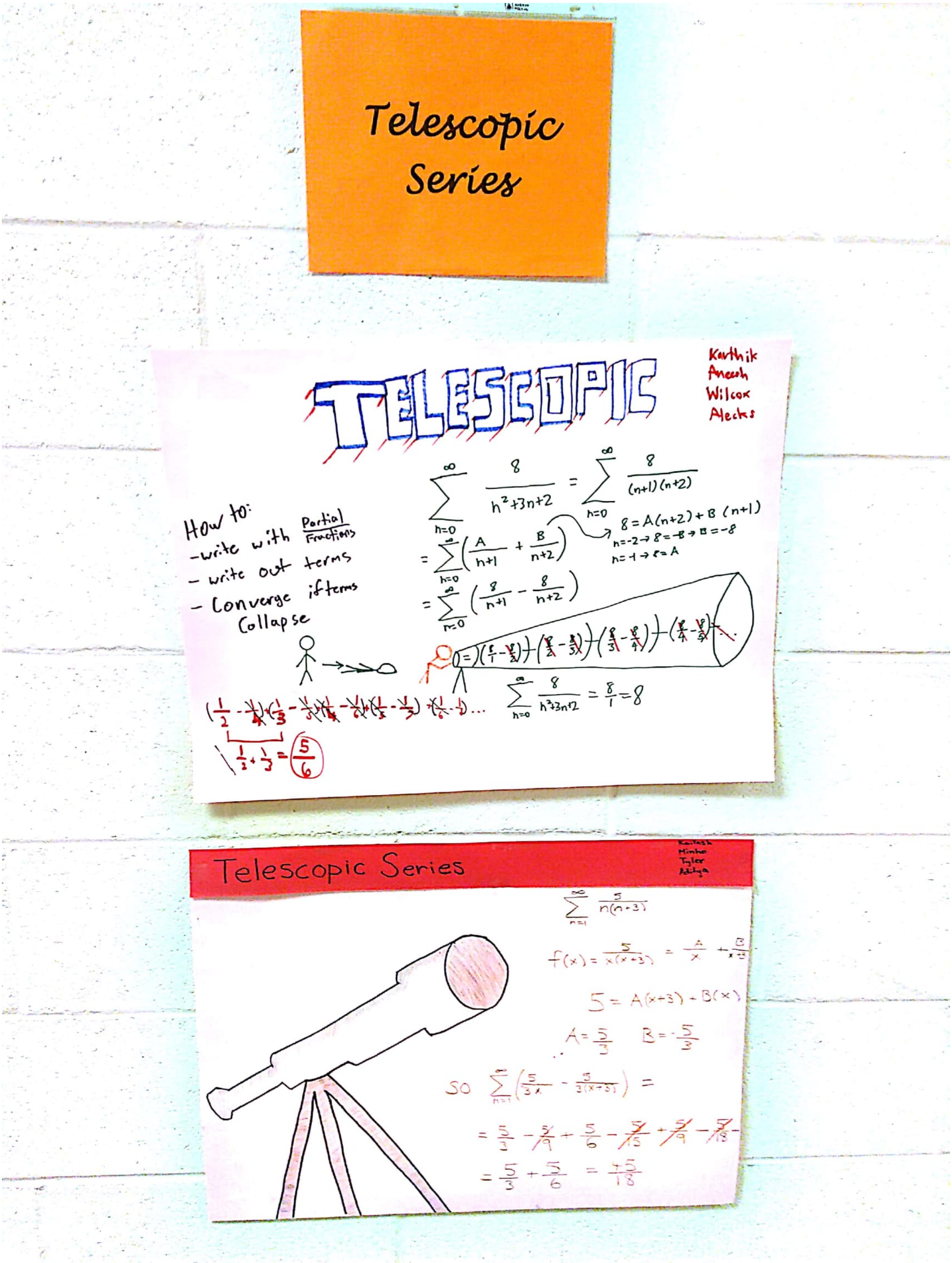
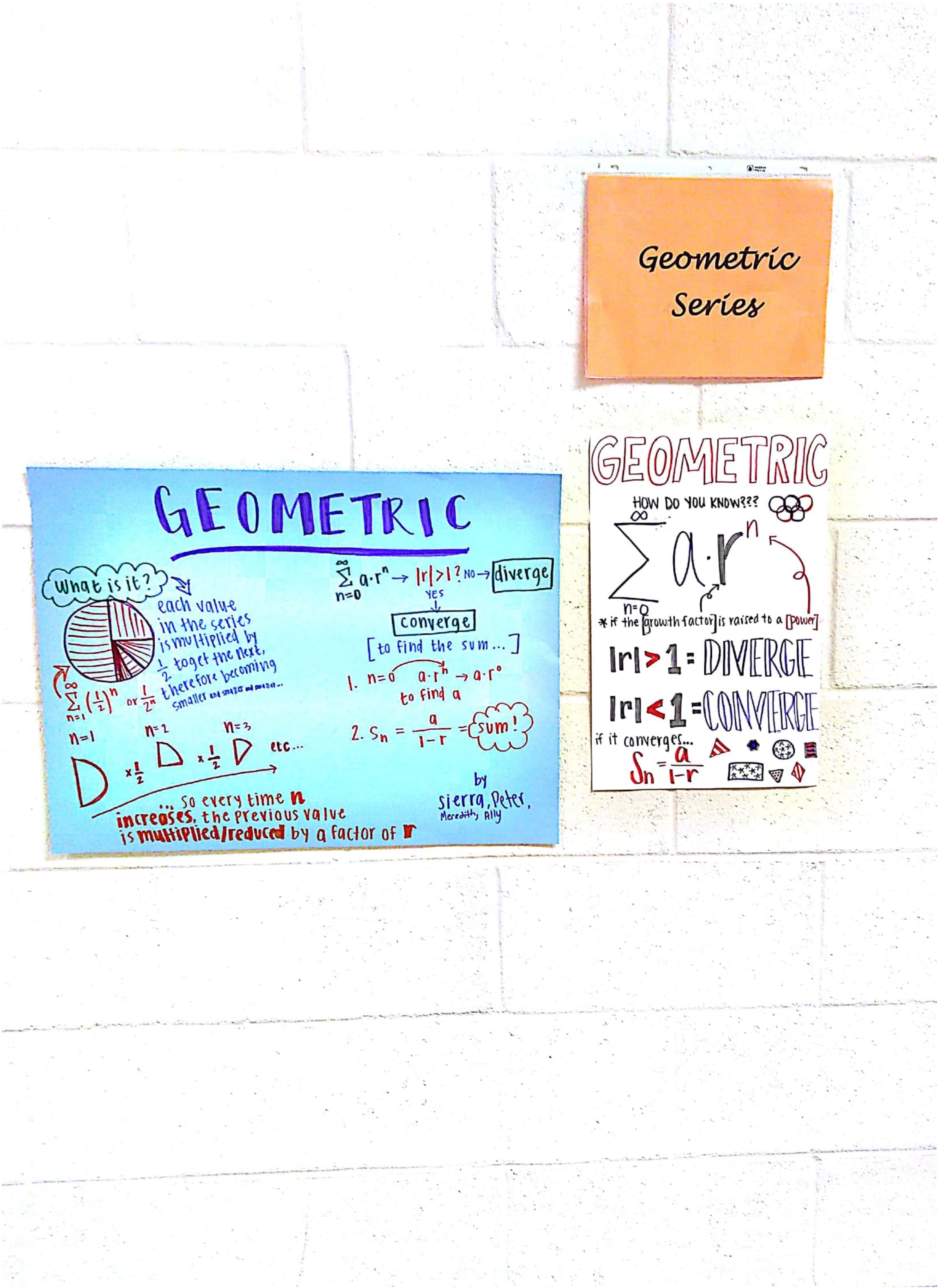
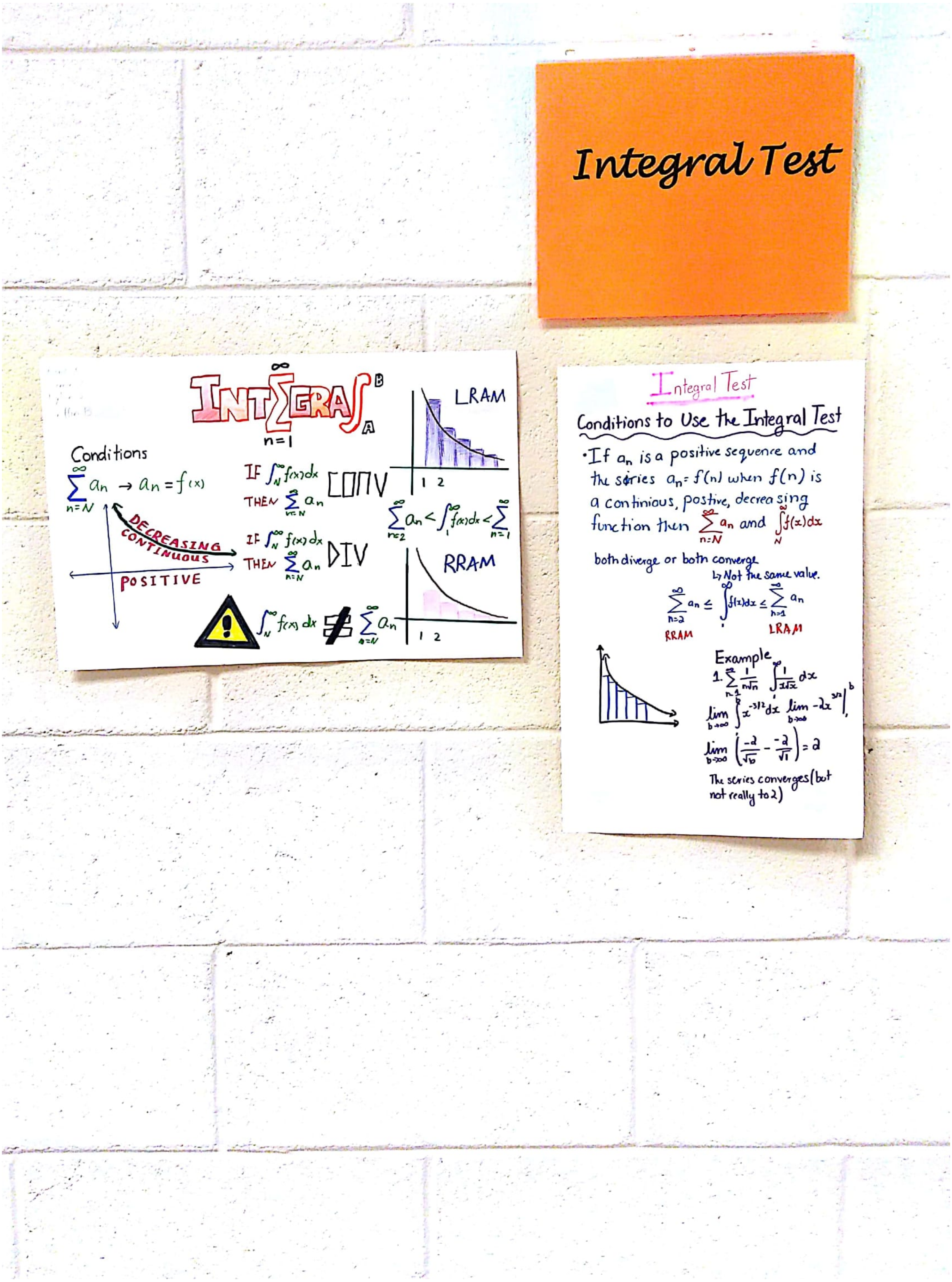
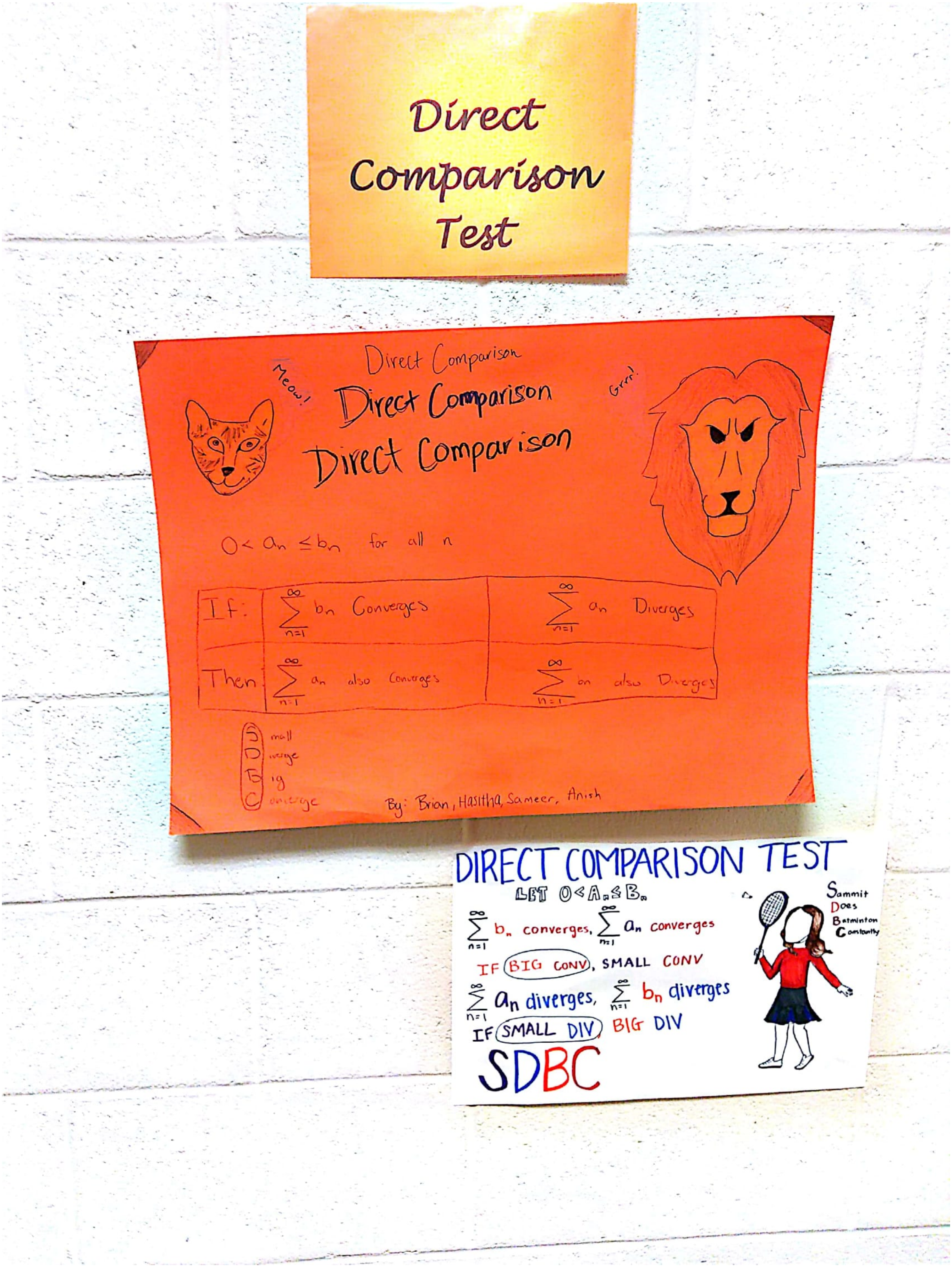

Office DEPOT
p-Series
$\sum_{n=1}^{\infty} \frac{1}{n^p}$ → Diverges if p is ≤ 1
P-SERIES
$\sum_{n=1}^{\infty} \frac{1}{n^P}$ → Converges if P is >1
TEST
Pea-Series
$\sum_{n=1}^{\infty} \frac{1}{n^P}$
$p>1$: Converges
$p\le 1$: Diverges
If there's one P, no one is fed and everyone DIVERGES.
If there's lots of P's, everyone is fed and they all CONVERGE.



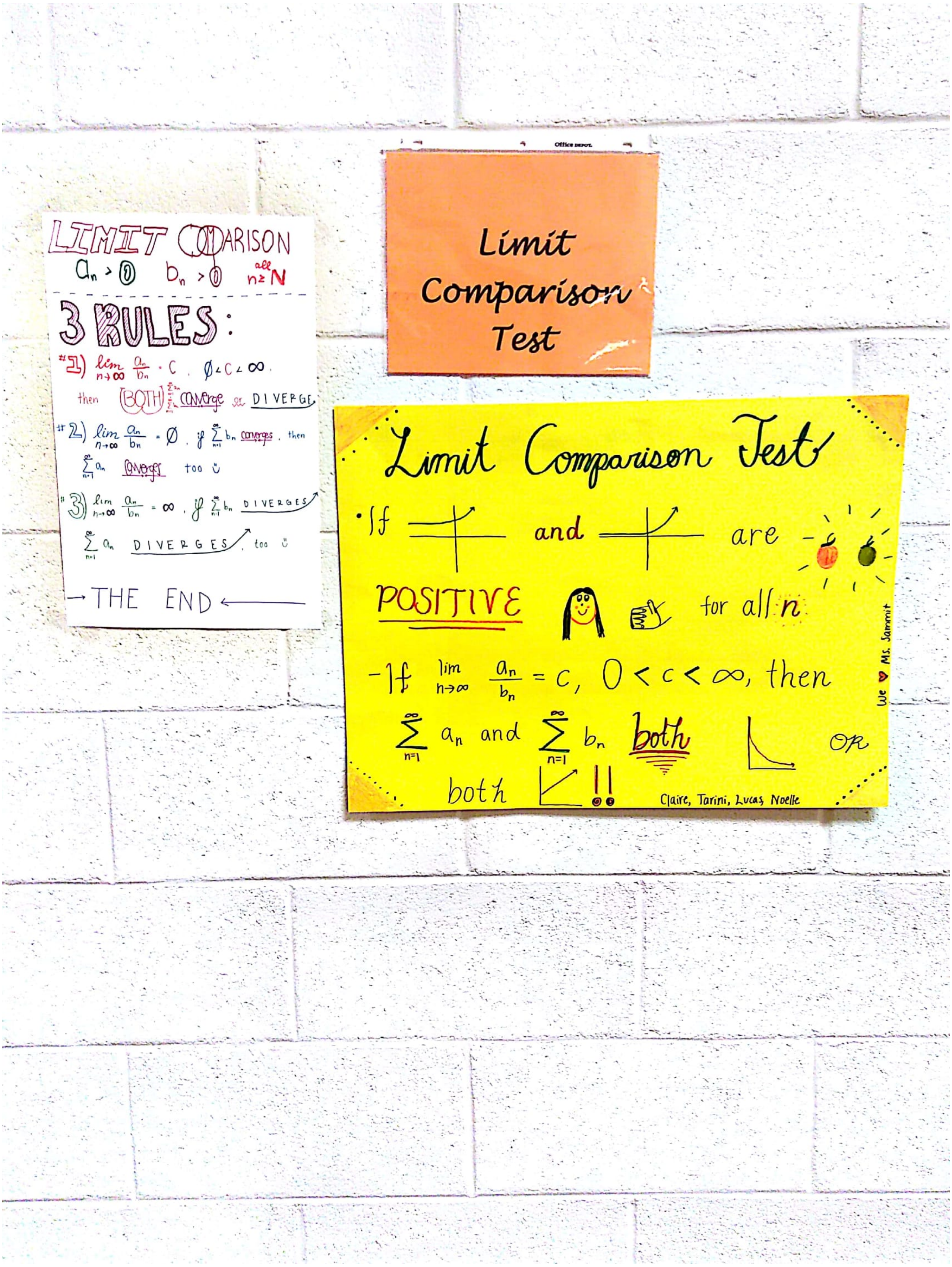
Telescopic Series
TELESCOPIC
Karthik
Wilcox
Alecks
How to:
-write with Partial Fractions
- write out terms
- Converge if terms Collapse
$\sum_{n=0}^{\infty} \frac{8}{n^2+3n+2} = \sum_{n=0}^{\infty} \frac{8}{(n+1)(n+2)}$
$8 = A(n+2) + B(n+1)$
$n=-2 \to 8 = -B \to B = -8$
$n=-1 \to 8 = A$
$= \sum_{n=0}^{\infty} \left(\frac{A}{n+1} + \frac{B}{n+2}\right)$
$= \sum_{n=0}^{\infty} \left(\frac{8}{n+1} - \frac{8}{n+2}\right)$
$\sum_{n=0}^{\infty} \frac{8}{n^2+3n+2} = \frac{8}{1} = 8$
$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$
Telescopic Series
$\sum_{n=1}^{\infty} \frac{5}{n(n+3)}$
$f(x) = \frac{5}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$
$5 = A(x+3) + B(x)$
$A = \frac{5}{3}$ $B = -\frac{5}{3}$
$\text{so } \sum_{n=1}^{\infty} \left(\frac{5}{3x} - \frac{5}{3(x+3)}\right) =$
$= \frac{5}{3} - \frac{5}{9} + \frac{5}{6} - \frac{5}{15} + \frac{5}{9} - \frac{5}{18}$
$= \frac{5}{3} + \frac{5}{6} = \frac{45}{18}$

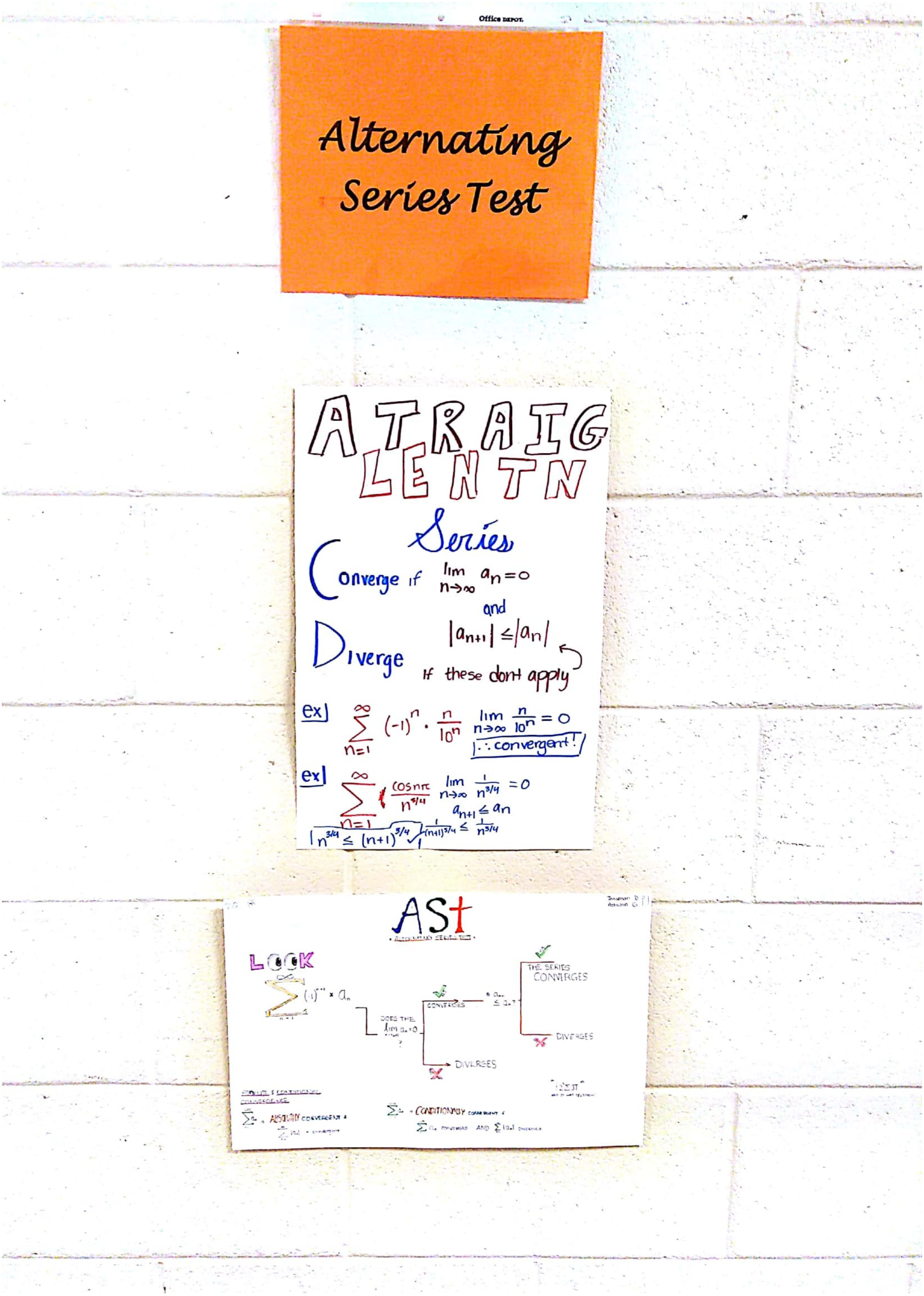

Geometric Series
GEOMETRIC
What is it?
each value in the series is multiplied by 1/2 to get the next, therefore becoming smaller and smaller
n=1
n=2
n=3
etc...
... so every time n increases, the previous value is multiplied/reduced by a factor of r
diverge
YES
converge
[to find the sum...]
1. n=0
to find a
2. Sn = a / 1-r = sum!
by
Sierra, Peter, Meredith, Ally
GEOMETRIC
HOW DO YOU KNOW???
*if the growth factor is raised to a power
|r|>1 = DIVERGE
|r|<1 = CONVERGE
if it converges...
Sn = a / 1-r

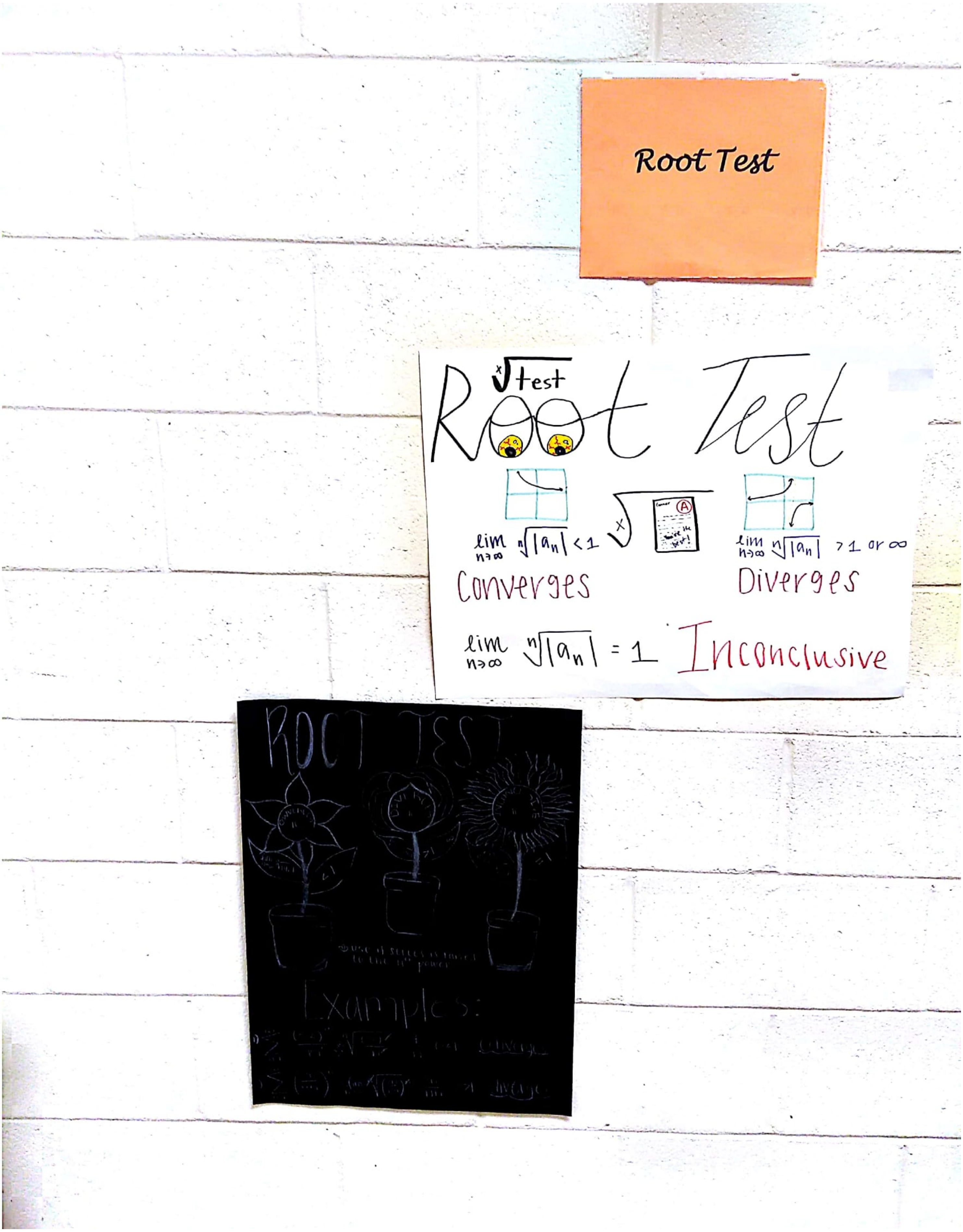

Integral Test
INTΣGRAL
n=1
Conditions
$\sum_{n=N}^{\infty} a_n \to a_n = f(x)$
DECREASING
CONTINUOUS
POSITIVE
IF $\int_N^{\infty} f(x)dx$ CONV THEN $\sum_{n=N}^{\infty} a_n$
IF $\int_N^{\infty} f(x)dx$ DIV THEN $\sum_{n=N}^{\infty} a_n$
$\int_N^{\infty} f(x)\,dx \neq \sum_{n=N}^{\infty} a_n$
LRAM
RRAM
1 2
$\sum_{n=2}^{\infty} a_n < \int_1^{\infty} f(x)dx < \sum_{n=1}^{\infty}$
Integral Test
Conditions to Use the Integral Test
•If a_n is a positive sequence and the series $a_n = f(n)$ when $f(n)$ is a continious, postive, decreasing function then $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)dx$
both diverge or both converge
↳ Not the same value.
$\sum_{n=2}^{\infty} a_n \leq \int_1^{\infty} f(x)dx \leq \sum_{n=1}^{\infty} a_n$
RRAM
LRAM
Example
1. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$ $\int_1^{\infty} \frac{1}{x\sqrt{x}} dx$
$\lim_{b\to\infty} \int_1^b x^{-3/2} dx$ $\lim_{b\to\infty} -2x^{-1/2} \Big|_1^b$
$\lim_{b\to\infty} \left(\frac{-2}{\sqrt{b}} - \frac{-2}{\sqrt{1}}\right) = 2$
The series converges (but not really to 2)

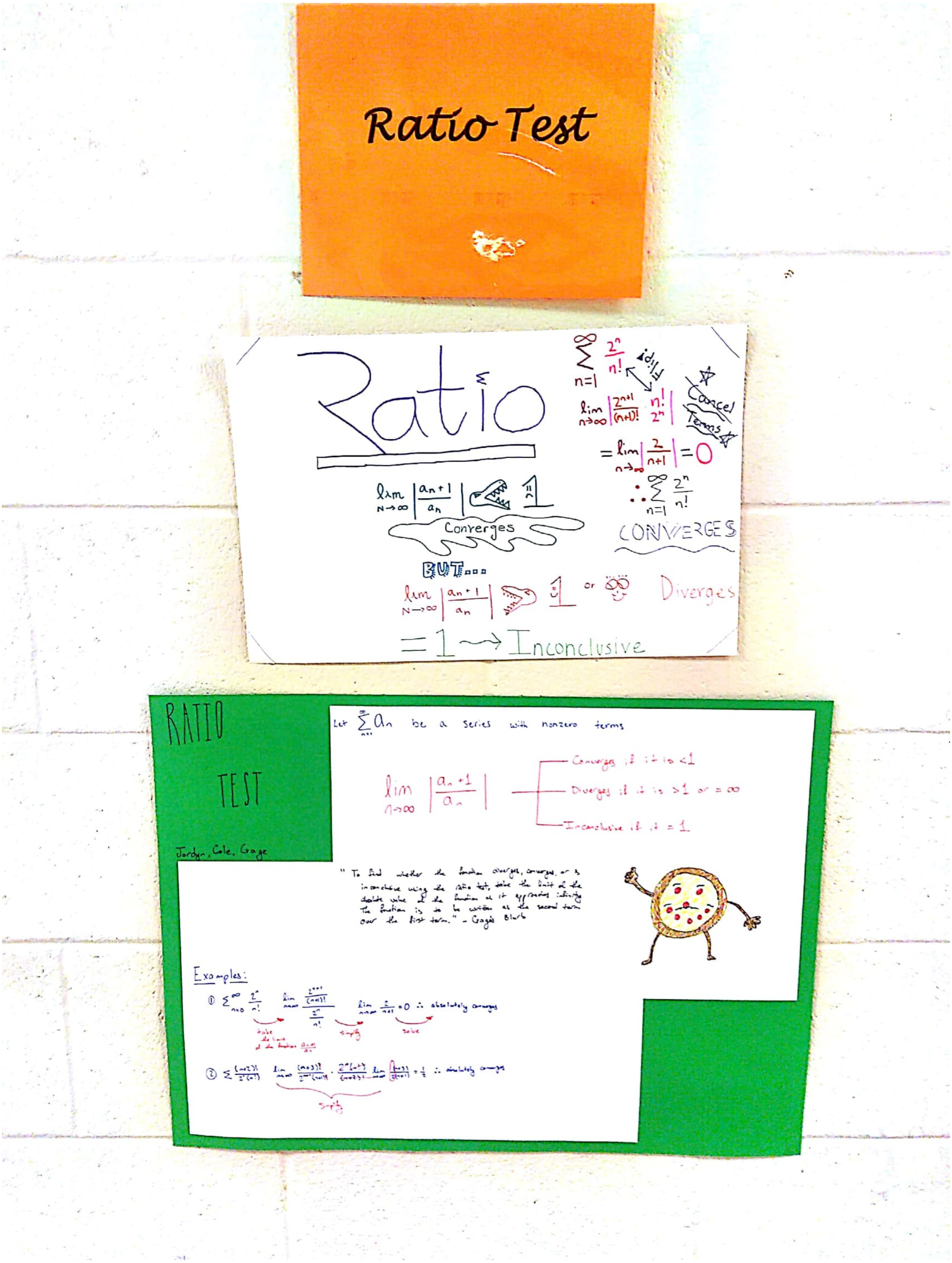


Direct Comparison Test
Direct Comparison
Direct Comparison
Direct Comparison
Meow!
Grrr!
$0 < a_n \le b_n$ for all n
If: $\sum_{n=1}^{\infty} b_n$ Converges
$\sum_{n=1}^{\infty} a_n$ Diverges
Then: $\sum_{n=1}^{\infty} a_n$ also Converges
$\sum_{n=1}^{\infty} b_n$ also Diverges
Small
Diverge
Big
Converge
By: Brian, Hasitha, Sameer, Anish
DIRECT COMPARISON TEST
LET $0 < A_n \le B_n$
$\sum_{n=1}^{\infty} b_n$ converges, $\sum_{n=1}^{\infty} a_n$ converges
IF BIG CONV, SMALL CONV
$\sum_{n=1}^{\infty} a_n$ diverges, $\sum_{n=1}^{\infty} b_n$ diverges
IF SMALL DIV, BIG DIV
SDBC
Sammit Does Batminton Constantly


LIMIT COMPARISON
all
3 RULES:
then BOTH converge or DIVERGE
converges, then
converges too
DIVERGES
DIVERGES too
→ THE END ←
Limit Comparison Test
Limit Comparison Test
If and are
POSITIVE
for all n
If lim an/bn = c, 0 < c < ∞, then
an and bn both
OR
both
We ♥ Ms. Sammit
Claire, Tarini, Lucas, Noelle


Office DEPOT.
Alternating
Series Test
ATRAIG
LENTN
Series
Converge if $\lim_{n\to\infty} a_n = 0$
and
$|a_{n+1}| \leq |a_n|$
Diverge if these don't apply
ex) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{10^n}$ $\lim_{n\to\infty} \frac{n}{10^n} = 0$
$\therefore$ convergent!
ex) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/4}}$ $\lim_{n\to\infty} \frac{1}{n^{3/4}} = 0$
$a_{n+1} \leq a_n$
$n^{3/4} \leq (n+1)^{3/4}$ $\frac{1}{(n+1)^{3/4}} \leq \frac{1}{n^{3/4}}$
ASt
LOOK
$\sum_{n=1}^{\infty} (-1)^{n+1} \times a_n$
DOES THE $\lim a_n = 0$?
CONVERGES
DIVERGES
THE SERIES CONVERGES
DIVERGES
CONDITIONALLY
ABSOLUTELY CONVERGENT


Root Test
Root Test
test
$\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
Converges
$\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ or ∞
Diverges
$\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$ Inconclusive
ROOT TEST
Examples:



Ratio Test
Ratio
$\sum_{n=1}^{\infty} \frac{2^n}{n!}$
Flip!
Cancel Terms
$\lim_{n\to\infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right|$
$= \lim_{n\to\infty} \left| \frac{2}{n+1} \right| = 0$
$\therefore \sum_{n=1}^{\infty} \frac{2^n}{n!}$
CONVERGES
$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$
Converges
BUT...
$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| > 1$ or ∞ Diverges
$= 1 \rightsquigarrow$ Inconclusive
RATIO TEST
Jordyn, Cole, Gage
Let $\sum_{n=1}^{\infty} a_n$ be a series with nonzero terms
$\lim_{n\to\infty} \left| \frac{a_n+1}{a_n} \right|$
Converges if it is <1
Diverges if it is >1 or = ∞
Inconclusive if it = 1
" To find whether the function diverges, converges, or is inconclusive using the ratio test, take the limit of the absolute value of the function as it approaches infinity. The function is to be written as the second term over the first term." - Gage's Blurb
Examples: