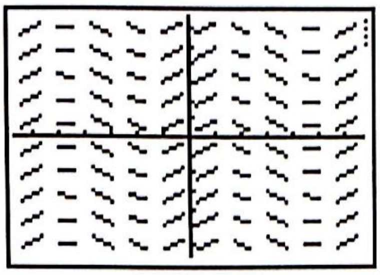
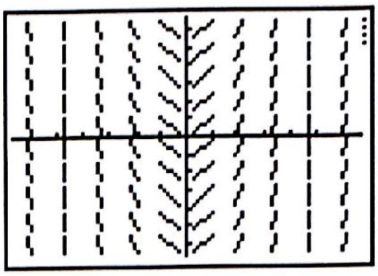
KEY

Below are six examples of **slope fields**. Match them with the correct differential equation. Explain each choice.

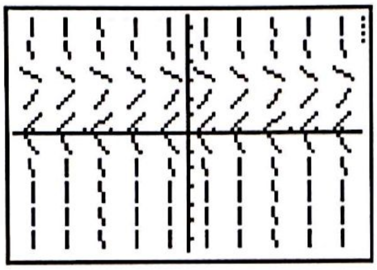
a.



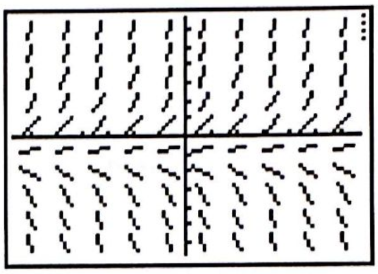
b.



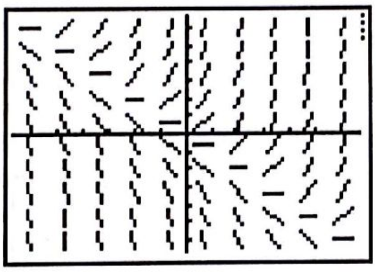
c.



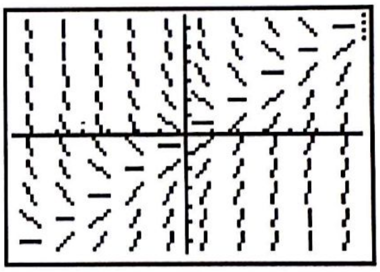
d.



e.



f.



1. $\frac{dy}{dx} = x - y$ f

2. $\frac{dy}{dx} = 1 + y$ d

3. $\frac{dy}{dx} = \cos x$ a

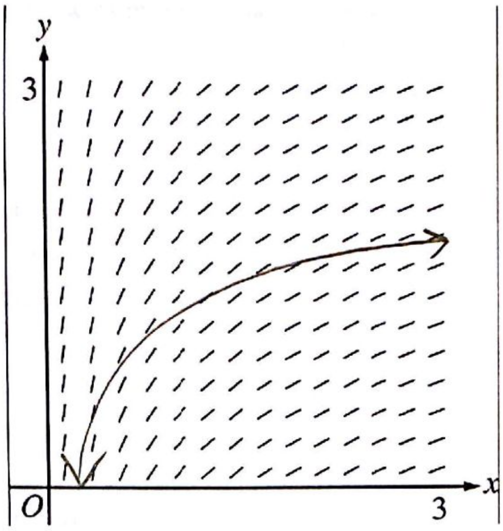
4. $\frac{dy}{dx} = 2x$ b

5. $\frac{dy}{dx} = x + y$ e

6. $\frac{dy}{dx} = y(3 - y)$ c

KEY

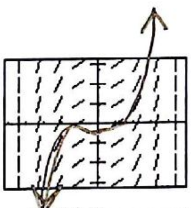
From the May 2008 *AP Calculus Course Description:*
15.



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y=x^2$ (B) $y=e^x$ (C) $y=e^{-x}$ (D) $y=\cos x$ (E) $y=\ln x$

16.



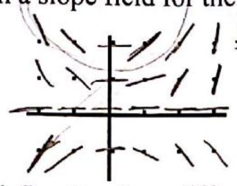
The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y=\sin x$ (B) $y=\cos x$ (C) $y=x^2$ (D) $y=\frac{1}{6}x^3$ (E) $y=\ln x$

KEY

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



1B) $\frac{dy}{dx} = \frac{(1)(1)}{2} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x-1)$

(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

$f(1.2) = \frac{1}{2}(1.2) + \frac{1}{2} = \frac{11}{10} = 1.1$

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part c

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

CC↑ so tan line is underestimate

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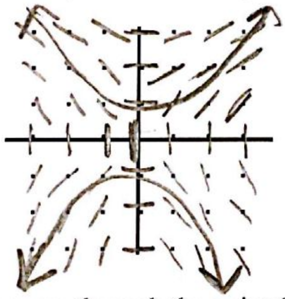
18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(A) On the axes provided, sketch a slope field for the given differential equation.



e) $y = -\sqrt{x^2+1}$

(B) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(D) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(E) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

17C) $\int \frac{2}{y}\,dy = \int x\,dx$

$2\ln|y| = \frac{x^2}{2} + C$

$2\ln 1 = \frac{1}{2} + C$

$0 = \frac{1}{2} + C$

$C = -\frac{1}{2}$

$2\ln|y| = \frac{x^2}{2} - \frac{1}{2}$

$\ln|y| = \frac{x^2}{4} - \frac{1}{4}$

$y = e^{x^2/4 - 1/4}$

$f(1.2) \approx 1.116$

18C) $\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$\frac{1}{2} = 0 + C$

$C = \frac{1}{2}$

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{1}{2}$

$y^2 = x^2 + 1$

$y = \pm\sqrt{x^2+1} \rightarrow$ $y = \sqrt{x^2+1}$

Kuta Software - Infinite Calculus

Name Key

Separable Differential Equations

Date______ Period____

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = e^{x-y}$

$e^x \cdot e^{-y}$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$\int e^y dy = \int e^x dx$

ln $e^y = \ln(e^x + c)$

$y = \ln(e^x + c)$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\int \sec^2 y \, dy = \int 1 \, dx$

$\tan y = x + c$

$y = \tan^{-1}(x + c)$

3) $\frac{dy}{dx} = xe^y$

$\frac{1}{e^y} dy = x\,dx$

$\int e^{-y} dy = \int x\,dx$

$-e^{-y} = \frac{x^2}{2} + c$

$\cancel{\ln e^{-y} = \ln -\frac{x^2}{2} + c}$

$-y = \ln\left(-\frac{x^2}{2}\right) + c$

$y = -\ln\left(-\frac{x^2}{2}\right) + c$

4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

$\int e^{2y} dy = \int 2x\,dx$

$\frac{e^{2y}}{2} = x^2 + C$

$\cancel{\ln e^{2y}} = \ln 2x^2 + C$

$2y = \ln(2x^2 + C)$

$y = \frac{\ln(2x^2 + C)}{2}$

5) $\frac{dy}{dx} = 2y - 1$

$\int \frac{1}{2y-1} = \int 1 dx$

$\frac{1}{2} \ln|2y-1| = x + C$

$\ln|2y-1| = 2x + C$

$e^{2x+c} = |2y-1|$

$e^{2x} \cdot e^c = |2y-1|$

$Ce^{2x} = 2y - 1$

$Ce^{2x} + 1 = 2y$

$\frac{Ce^{2x}+1}{2} = y$

6) $\frac{dy}{dx} = 2yx + yx^2$

$\frac{dy}{dx} = y(2x + x^2)$

$\frac{1}{y} dy = (2x + x^2) dx$

$\ln|y| = x^2 + \frac{x^3}{3} + C$

$e^{x^2 + x^3/3 + C} = |y|$

$y = e^{x^2 + x^3/3} \cdot e^c$

$y = Ce^{x^2 + x^3/3}$