Deriving/Integrating a Power Series:

If $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n =$

$a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \ldots$

has ROC $R > 0$, then on the interval $(c-R, c+R)$, f is differentiable ($\therefore$ continuous)

Also,

* 1) $f'(x) = \sum_{n=1}^{\infty} n a_n (x-c)^{n-1}$

2) $\int f(x)\,dx = C + \sum_{n=0}^{\infty} \frac{a_n (x-c)^{n+1}}{(n+1)}$

* ROC obtained by differentiating or integrating is same as original series

* IOC may be dif as a result of behavior at endpts

1) $f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-2)^n}{n}$

Find IOC:

a) $f(x)$ $\lim_{n\to\infty} \left| \frac{(-1)^{n+2}(x-2)^{n+1}}{(n+1)} \cdot \frac{n}{(-1)^{n+1}(x-2)^n} \right|$

$= \left| \frac{\cancel{(-1)^n}(-1)^2\cancel{(x-2)^n}(x-2)}{(n+1)} \cdot \frac{n}{\cancel{(-1)^n}(-1)^1\cancel{(x-2)^n}} \right| = \left| \frac{-\cancel{n}}{\cancel{(n+1)}} \cdot x-2 \right|$ -1

$|-1(x-2)| < 1$ $\quad -1 < -x+2 < 1$ $\quad -3 < -x < -1$ $\quad 3 > x > 1$ $\quad 1 < x < 3$ $\rightarrow$

* check endpts:

$x=1: \frac{(-1)^{n+1}(-1)^n}{n} = \frac{(-1)^n(-1)(-1)^n}{n} = \frac{-1}{n}$ di

$x=3: \frac{(-1)^{n+1}(1)^n}{n} = \frac{(-1)^n(-1)(1)^n}{n} = \frac{-(-1)^n}{n}$ con

$\boxed{1<x\leq 3}$

b) $f'(x) = (-1)^{n+1} \cdot \frac{\cancel{n}(x-2)^{n-1}}{\cancel{n}} = (-1)^{n+1}(x-2)^{n-1}$

$x=1: (-1)^{n+1}(-1)^{n-1} = (-1)^n(-1)(-1)^n(-1)^{-1} = 1$ di

$x=3: (-1)^{n+1}(1)^{n-1} = (-1)^n\cancel{(-1)}\cancel{(1)^n}\cancel{(1)^{-1}} =$ di

$-1(+1)^n$

$\boxed{1<x<3}$

geo

c) $\int f(x) = \frac{(-1)^{n+1}(x-2)^{n+1}}{n(n+1)}$

$\frac{1}{n^2+n} < \frac{1}{n^2}$
cor

$x=1: \frac{(-1)^{n+1}(-1)^{n+1}}{n(n+1)} = \frac{1^n}{n^2+n} = \frac{1}{n^2+n}$ con

$x=3: \frac{(-1)^{n+1}(1)^{n+1}}{n(n+1)} = \frac{(-1)^n(-1)\cancel{(1^n)}(1)}{n^2+n} = \frac{-(-1)^n}{n^2+n}$

con

$\boxed{1\leq x\leq 3}$

② $f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{(x-5)^n}{n \cdot 5^n}$

Find IOC:

a) $f(x)$ $\lim_{n \to \infty} \left| \dfrac{(-1)^{n+2} (x-5)^{n+1}}{(n+1) \cdot 5^{n+1}} \cdot \dfrac{n \cdot 5^n}{(-1)^{n+1} (x-5)^n} \right|$

$= \lim_{n \to \infty} \left| \dfrac{\cancel{(-1)^n} \cdot \cancel{(-1)^2} \cancel{(x-5)^n} (x-5)}{(n+1) \cdot \cancel{5^n} \cdot 5} \cdot \dfrac{n \cdot \cancel{5^n}}{\cancel{(-1)^n} (-1) \cancel{(x-5)^n}} \right|$

$= \lim_{n \to \infty} \left| \dfrac{\cancel{n} (x-5)}{(\cancel{n}+1) \cdot -5} \right| = \left| \dfrac{-(x-5)}{5} \right| < 1$

$-1 < \dfrac{-x+5}{5} < 1$

$-5 < -x+5 < 5$

$-10 < -x < 0$

$10 > x > 0$

$\boxed{0 < x \leq 10}$

* check endpts:

$x = 0$: $(-1)^{n+1} \dfrac{(-5)^n}{n \cdot 5^n}$

$= \dfrac{\cancel{(-1)^n} (-1) (-1)^n \cdot \cancel{5^n}}{n \cdot \cancel{5^n}} = \dfrac{-1}{n}$ di

$x = 10$: $\dfrac{(-1)^{n+1} \cancel{(5)^n}}{n \cdot \cancel{5^n}}$ con

AST $\dfrac{1}{n}$ di

$\lim_{n \to \infty} \dfrac{(-1)^{n+1}}{n} = 0$ cond con

b) $f'(x) = \cancel{\sum_{n=1}^{\infty}} \frac{(-1)^{n+1} \cdot \cancel{n} \cdot (x-5)^{n-1}}{\cancel{n} \cdot 5^n} = \frac{(-1)^{n+1}(x-5)^{n-1}}{5^n}$

$x=0: \frac{(-1)^{n+1}(-5)^{n-1}}{5^n} = \frac{(-1)^n(-1)(-5)^n(-5)^{-1}}{5^n}$

$= \frac{\cancel{(-1)^n}(-1)\cancel{(-1)^n}\cancel{(5)^n}}{-5 \cdot \cancel{5^n}} = \frac{1}{5}$ di

$x=10: \frac{(-1)^{n+1}(5)^{n-1}}{5^n} = \frac{(-1)^n(-1) \cdot \cancel{5^n} \cdot 5^{-1}}{\cancel{5^n}}$

$= -\frac{(-1)^n}{5}$ di

$\boxed{0 < x < 10}$

c) $\int f(x)dx = \frac{(-1)^{n+1}}{n \cdot 5^n} \cdot \frac{(x-5)^{n+1}}{(n+1)}$

$\frac{5}{n^2+n}$ small con $\frac{5}{n^2}$ big con

$x=0: \frac{\cancel{(-1)^n}(-1)^1 \cdot \cancel{(-5)^n}(-5)}{n \cdot \cancel{5^n}(n+1)} = \frac{5}{n(n+1)}$ con

$x=10: \frac{(-1)^{n+1}}{n \cdot 5^n} \cdot \frac{5^{n+1}}{n+1} = \frac{(-1)^n(-1)\cancel{(5^n)}(5)}{n \cdot \cancel{5^n}(n+1)}$

$= \frac{-5(-1)^n}{n(n+1)}$ con

$\boxed{0 \le x \le 10}$