

Derivatives Worksheet

In problems 1 – 14, find the derivative:

1. $f(x) = \frac{x^2 - 3}{2x - 5}$

3. $y = \tan(\ln(2x + 1))$

5. $y = \frac{\cos x}{1 + \sin x}$

7. $f(x) = \ln(1 + e^x)$

9. $y = \left(\frac{1 + \sin x}{1 - \cos x}\right)^2$

11. $y = x\sqrt{2x + 1}$

13. $y = \sec x \tan x$

2. $f(x) = e^x \cos x$

4. $y = \cos^4(x^2)$

8. $f(x) = e^{-\frac{x^2}{2}}$

10. $y = \sin\left(\frac{2}{x}\right)$

12. $y = 2x^{3.5} + x^{-3.5} + \tan(3.5x) + 3.5^2$

14. $y = \sqrt{1 + \cos x}$

15. Find the derivative of $y = \frac{fg}{h}$, in terms of $f, g, h, f', g',$ and h' , where $f, g,$ and h are functions of x .

16. Find the rate of change of $y = (4x^3 + 7x^2 + 1)^2$ at $x = -1$.

17. If $f(x) = \sin^3 x$, find $f''(x)$.

18. Find the slope of the line tangent to the curve $y = \cos(2x)$ at the point where $x = \frac{\pi}{6}$.

19. Let $f(x) = \begin{cases} \sin(\pi x), & 0 \leq x \leq 1 \\ ax + b, & 1 < x \leq 2 \end{cases}$. Find the values of a and b such that $f(x)$ is differentiable at $x = 1$.

20. Which of the following functions are NOT differentiable at $x = 0$?

I. $y = \sqrt{4 - x^2}$ II. $y = x^{\frac{2}{3}}$ III. $y = x^{\frac{4}{3}}$ IV. $y = x^{-2}$ V. $y = |\sin(2x)|$

Worksheet Answers and Solutions

$$1. \frac{(2x-5)2x - (x^2-3)2}{(2x-5)^2} = \frac{2x^2 - 10x + 6}{(2x-5)^2}$$

$$2. e^x (\cos x - \sin x)$$

$$3. \frac{2 \sec^2(\ln(2x+1))}{2x+1}$$

$$4. 4 \cos^3(x^2) \cdot (-\sin(x^2)) \cdot 2x = -8x \cos^3(x^2) \sin(x^2)$$

$$5. \frac{(1+\sin x)(-\sin x) - \cos x \cdot \cos x}{(1+\sin x)^2} = -\frac{1+\sin x}{(1+\sin x)^2} = -\frac{1}{1+\sin x}$$

$$6. \frac{6x}{1+9x^4}$$

$$7. \frac{e^x}{1+e^x}$$

$$8. -xe^{-\frac{x^2}{2}}$$

$$9. \frac{2(1+\sin x)}{1-\cos x} \cdot \frac{(1-\cos x)\cos x - (1+\sin x)\sin x}{(1-\cos x)^2} = \frac{2(1+\sin x)(\cos x - \sin x - 1)}{(1-\cos x)^3}$$

$$10. -\frac{2}{x^2} \cos\left(\frac{2}{x}\right)$$

$$11. \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$$

$$12. 7x^{2.5} - 3.5x^{-4.5} + 3.5 \sec^2(3.5x)$$

$$13. \sec x (\sec^2 x + \tan^2 x)$$

$$14. \frac{-\sin x}{2\sqrt{1+\cos x}}$$

$$15. \frac{f'gh + fg'h - fgh'}{h^2}$$

$$16. 2(4x^3 + 7x^2 + 1)(12x^2 + 14x) \Big|_{x=-1} = -16$$

17. $\frac{d}{dx} [3 \sin^2 x \cos x] = 3 [-\sin^3 x + 2 \sin x \cos^2 x] = 3 \sin x (2 \cos^2 x - \sin^2 x)$

18. $-2 \sin(2x) \Big|_{x=\frac{\pi}{6}} = -\sqrt{3}$

19. $\sin \pi = a + b; \pi \cos \pi = a \Rightarrow a = -\pi, b = \pi$

20. II, IV, and V

21. Continuous but not differentiable

22. $2yy' = -\frac{x^2}{2} \Rightarrow y' = -\frac{yx^2}{1} \Rightarrow y'' = \frac{x^2 y' + 2yx}{-2yx^2 + 2yx} = \frac{-\frac{x^2}{2} + 2yx}{2y^2 x^2 - 1} = \frac{y^2 x^4}{y^3 x^4}$

23. $n^n e^{nx}$

24. $3x^2 + 2y + 2xy' + 3y^2 y' = 0 \Rightarrow y' = -\frac{2x + 3y^2}{3x^2 + 2y} \Rightarrow y' \Big|_{x=-1, y=2} = -\frac{10}{7}$

25. $f(c) = 0 \Rightarrow \frac{1}{c^2} = 4 \Rightarrow f'(c) = 4 + \frac{1}{c^2} = 8 \Rightarrow g'(0) = \frac{f'(c)}{1} = \frac{8}{1}$

26. $y' = 1 + \cos(xy) \Rightarrow y' = \frac{1 + y \cos(xy)}{1 + y \cos(xy)}$

27. $g'(7) = \frac{1}{1} = \frac{6}{1}$