Derivatives and Integrals of Known Series: (3 terms and general term)

① $f(x) = e^x$ $\quad 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ $\quad \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$f'(x)$ $\quad 1 + \frac{2x}{2!} + \frac{3x^2}{3!}$ $\quad \sum_{n=0}^{\infty} \frac{n \cdot x^{n-1}}{n!}$

$\int f(x)dx$ $\quad x + \frac{x^2}{2} + \frac{x^3}{3 \cdot 2!}$ $\quad \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)n!}$

② $f(x) = \frac{1}{1-x}$ $\quad 1 + x + x^2 + x^3 + x^4$ $\quad \sum_{n=0}^{\infty} x^n$

$f'(x)$ $\quad 1 + 2x + 3x^2$ $\quad \sum_{n=0}^{\infty} n \cdot x^{n-1}$

$\int f(x)dx$ $\quad x + \frac{x^2}{2} + \frac{x^3}{3}$ $\quad \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}$

③ $f(x) = \sin x$ $\quad x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$f'(x)$ $\quad 1 - \frac{3x^2}{3!} + \frac{5x^4}{5!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n (2n+1) \cdot x^{2n}}{(2n+1)!}$

$\int f(x)\,dx$ $\quad \frac{x^2}{2} - \frac{x^4}{4 \cdot 3!} + \frac{x^6}{6 \cdot 5!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+2}}{(2n+2)(2n+1)!}$

④ $f(x) = \cos x$ $\quad 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n)!}$

$f'(x)$ $\quad -\frac{2x}{2!} + \frac{4x^3}{4!} - \frac{6x^5}{6!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2n \cdot x^{2n-1}}{(2n)!}$

$\int f(x)dx$ $\quad x - \frac{x^3}{3 \cdot 2!} + \frac{x^5}{5 \cdot 4!}$ $\quad \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)(2n)!}$