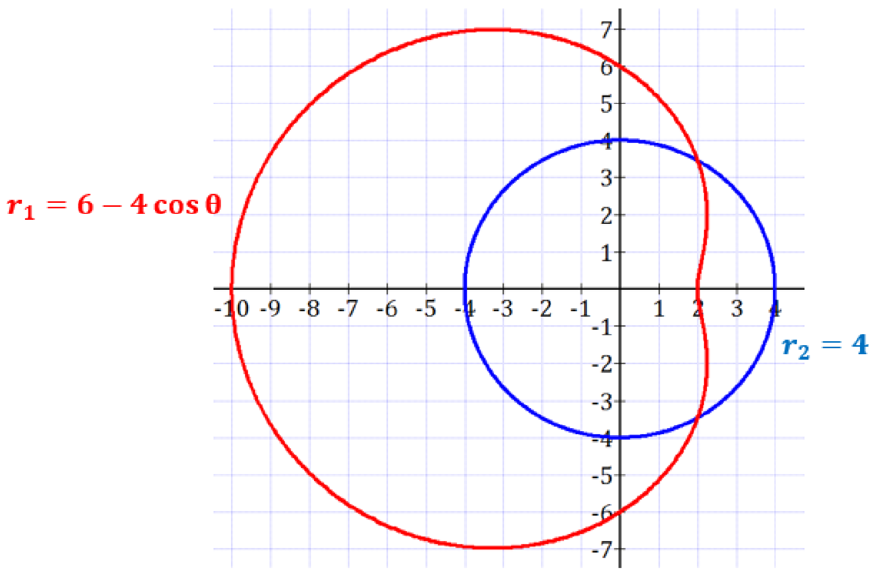
Name________________________________

Seat # ________ Date ____________________

Derivatives and Equations in Polar Coordinates

1) The graphs of the polar curves $r_1 = 6 - 4\cos\theta$ and $r_2 = 4$ are shown below. Enter both graphs in your graphing calculator and "explore" them.



2) Find the coordinates of the points of intersection of both curves for $0 \le \theta < 2\pi$. Write your answers using polar coordinates.

3) Find $\dfrac{dr_1}{d\theta}$ and $\dfrac{dr_2}{d\theta}$ in terms of θ.

Find $\left.\dfrac{dr_1}{d\theta}\right|_{\theta=\frac{\pi}{3}}$ and $\left.\dfrac{dr_2}{d\theta}\right|_{\theta=\frac{\pi}{3}}$. Interpret the meaning of your answers in the context of the problem.

4) A particle moves along the curve $r_1 = 6 - 4\cos\theta$ such that $\theta = -2t$. Find $\left.\frac{dr_1}{dt}\right|_{t=1}$, both by hand and efficiently using your graphing calculator. Interpret the meaning of your answer in the context of the problem.

5) Write an expression for $y_1(\theta)$, the y-coordinate of each point on the graph of $r_1 = 6 - 4\cos\theta$. For $0 \leq \theta < \frac{\pi}{2}$, there is one point on r_1 with y-coordinate equal to 4. Find the subject point. Express your answer using both rectangular and polar coordinates.

6) Write in terms of x and y an equation for the line tangent to the graph of the curve r_1 at the point where $\theta = \frac{\pi}{3}$.

7) Recall that a particle moves along the curve $r_1 = 6 - 4\cos\theta$ such that $\theta = -2t$. Find the position vector of the particle $\langle x(t), y(t)\rangle$ in terms of t. Use your calculator to find the velocity vector of the particle at $t = 1$.

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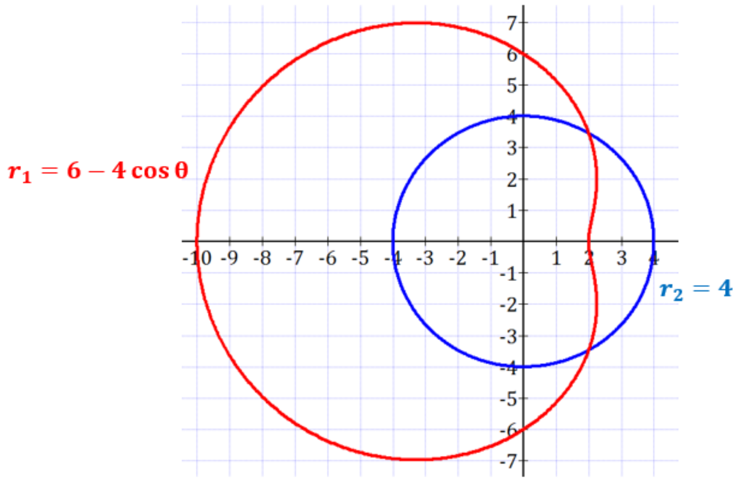
1) The graphs of the polar curves $r_1 = 6 - 4\cos\theta$ and $r_2 = 4$ are shown below. Enter both graphs in your graphing calculator and "explore" them.



2) Find the coordinates of the points of intersection of both curves for $0 \le \theta < 2\pi$. Write your answers using polar coordinates.

Collision points: $r_1 = r_2 \rightarrow 6 - 4\cos\theta = 4$

$$\theta = \frac{\pi}{3} \quad \text{or} \quad \frac{5\pi}{3}$$

For both values of θ, we have $r = 4$.

Collision points are $\left(4, \frac{\pi}{3}\right)$ and $\left(4, \frac{5\pi}{3}\right)$

3) Find $\frac{dr_1}{d\theta}$ and $\frac{dr_2}{d\theta}$ in terms of θ.

Find $\left.\frac{dr_1}{d\theta}\right|_{\theta=\frac{\pi}{3}}$ and $\left.\frac{dr_2}{d\theta}\right|_{\theta=\frac{\pi}{3}}$. Interpret the meaning of your answers in the context of the problem.

$$\frac{dr_1}{d\theta} = 4\sin\theta$$

$$\frac{dr_2}{d\theta} = 0$$

$$\left.\frac{dr_1}{d\theta}\right|_{\theta=\frac{\pi}{3}} = 4\sin\frac{\pi}{3} = 2\sqrt{3}$$

When we trace the graph of r_1 at $\theta = \frac{\pi}{3}$ the distance from the graph to the pole (origin) is increasing at the rate $2\sqrt{3}$.

$$\left.\frac{dr_2}{d\theta}\right|_{\theta=\frac{\pi}{3}} = 0$$

When we trace the graph of r_2 at $\theta = \frac{\pi}{3}$ the distance from the graph to the pole (origin) is not changing.

4) A particle moves along the curve $r_1 = 6 - 4\cos\theta$ such that $\theta = -2t$. Find $\left.\frac{dr_1}{dt}\right|_{t=1}$, both by hand and efficiently using your graphing calculator. Interpret the meaning of your answer in the context of the problem.

$$r_1 = 6 - 4\cos(-2t)$$

By hand: $\frac{dr_1}{dt} = -8\sin(-2t)$, so $\left.\frac{dr_1}{dt}\right|_{t=1} = -8\sin(-2)$

Using a calculator: $\left.\frac{d(6-4\cos(-2t))}{dt}\right|_{t=1} = 7.274$

Using a calculator the problem can also be done as

$$\left.\frac{dr_1}{dt}\right|_{t=1} = \left.\frac{dr_1}{d\theta}\right|_{\theta=-2} \cdot \left.\frac{d\theta}{dt}\right|_{t=1} = \left.\frac{d(6-4\cos\theta)}{d\theta}\right|_{\theta=-2} \cdot (-2) = (-3.637)(-2) = 7.274$$

5) Write an expression for $y_1(\theta)$, the y-coordinate of each point on the graph of $r_1 = 6 - 4\cos\theta$. For $0 \le \theta < \frac{\pi}{2}$, there is one point on r_1 with y-coordinate equal to 4. Find the subject point. Express your answer using both rectangular and polar coordinates.

$$y_1 = r_1 \cdot \sin\theta = (6 - 4\cos\theta)(\sin\theta) = 6\sin\theta - 4\cos\theta\sin\theta$$

$y_1 = 4 = 6\sin\theta - 4\cos\theta\sin\theta$ →use your calculator in "function mode" to get θ =1.1528

Store this value to obtain $r_1(1.1528) = 4.3766$.

Polar coordinates: r_1 =4.3766 and θ =1.1528

For rectangular coordinates:

$$x_1 = r_1\cos\theta \rightarrow x_1(1.1528) = 1.776$$

$$y_1 = r_1\sin\theta \rightarrow y_1(1.1528) = 4$$

6) Write in terms of x and y an equation for the line tangent to the graph of the curve r_1 at the point where $\theta = \frac{\pi}{3}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d(r_1\sin\theta)}{d\theta}}{\frac{d(r_1\cos\theta)}{d\theta}} = \frac{4\sin\theta\sin\theta + (6-4\cos\theta)\cos\theta}{4\sin\theta\cos\theta - (6-4\cos\theta)\sin\theta}$$

At $\theta = \frac{\pi}{3}$ we have:

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = -\frac{5}{\sqrt{3}} \quad \text{and} \quad x_1\left(\frac{\pi}{3}\right) = 2 \quad \text{and} \quad y_1\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

Tangent line: $y - 2\sqrt{3} = -\frac{5}{\sqrt{3}}(x - 2)$

7) Recall that a particle moves along the curve $r_1 = 6 - 4\cos\theta$ such that $\theta = -2t$. Find the position vector of the particle $\langle x(t), y(t)\rangle$ in terms of t. Use your calculator to find the velocity vector of the particle at $t = 1$.

Using $x_1 = r_1\cos\theta$ and $y_1 = r_1\sin\theta$ and $\theta = -2t$:

position vector: $\langle(6 - 4\cos(-2t))\cos(-2t), (6 - 4\cos(-2t))\sin(-2t)\rangle$

Using a calculator, velocity vector at $t = 1$ is

$$\left\langle \left.\frac{dx_1}{dt}\right|_{t=1}, \left.\frac{dy_1}{dt}\right|_{t=1} \right\rangle = \langle -16.9659, -0.235\rangle$$