Day 3 HW:

① a) $\sum_{n=1}^{\infty} \frac{(-1)^n(x-3)^n}{n}$ $\lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-3)^{n+1}}{n+1}\cdot\frac{n}{(-1)^n(x-3)^n}\right|$

$\lim_{n\to\infty}\left|\frac{\cancel{(-1)^n}(-1)\cancel{(x-3)^n}(x-3)^1}{(n+1)}\cdot\frac{n}{\cancel{(-1)^n}\cancel{(x-3)^n}}\right| \to \lim_{n\to\infty}\left|\frac{n}{n+1}\cdot -1(x-3)\right|$

$-1 < -x+3 < 1$

$-4 < -x < -2$

$4 > x > 2$

IOC: $2 < x \le 4$

Endpts:

$x=2$: $\frac{(-1)^n(-1)^n}{n} = \frac{(-1)^{2n}}{n} = \frac{1}{n}$ div p-series

$x=4$: $\frac{(-1)^n(1)^n}{n} = \frac{(-1)^n}{n}$ conv AST

b) $f'(x) = \frac{(-1)^n\cdot \cancel{n}(x-3)^{n-1}}{\cancel{n}} = (-1)^n(x-3)^{n-1}$

Endpts:

$x=2$: $(-1)^n(-1)^{n-1} = (-1)^n(-1)^n(-1)^{-1} = -(-1)^{2n} = -1$ div

$x=4$: $(-1)^n(1)^{n-1} = (-1)^n(1)^n(1)^{-1} = (-1)^n$ div

IOC: $2 < x < 4$

c) $\int f(x)\,dx = \frac{(-1)^n(x-3)^{n+1}}{n(n+1)}$ IOC: $[2,4]$

Endpts:

$x=2$: $\frac{(-1)^n(-1)^{n+1}}{n(n+1)} = \frac{(-1)^n(-1)^n(-1)^1}{n(n+1)} = \frac{(-1)^{2n}(-1)}{n(n+1)} = \frac{-1}{n^2+n}$

$\int_1^\infty \frac{-1}{x^2+x}\,dx$ $\frac{-1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ conv by integral or telescopic

$-1 = A(x+1) + Bx$

$x=-1$: $-1 = B(-1)$ $B=1$

$x=0$: $-1 = A(1)$ $A=-1$

$\lim_{b\to\infty}\int_1^b\left(\frac{1}{x+1}-\frac{1}{x}\right)dx = \ln|x+1| - \ln|x| = \ln\left|\frac{x+1}{x}\right| = 0$ conv

$x=4$: $\frac{(-1)^n(1)^{n+1}}{n(n+1)} = \frac{(-1)^n\cdot 1}{n(n+1)}$ conv by AST

2 a) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x}{(n+1)\cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{x^n}} \right| \to \lim_{n\to\infty} \left| \frac{1}{n+1} \cdot x \right|$

$0 \cdot |x| < 1$ always converges

IOC: $(-\infty, \infty)$

b) $f'(x) = \frac{n \cdot x^{n-1}}{n!} = \frac{\cancel{n} \cdot x^{n-1}}{\cancel{n}(n-1)!} = \frac{x^{n-1}}{(n-1)!}$

IOC: $(-\infty, \infty)$

c) $\int f(x)dx = \frac{x^{n+1}}{(n+1)\cdot n!}$ IOC: $(-\infty, \infty)$

3 a) $\sum_{n=0}^{\infty} \frac{(x+4)^n}{3^n(n+1)}$ $\lim_{n\to\infty} \left| \frac{(x+4)^{n+1}}{3^{n+1}(n+2)} \cdot \frac{3^n(n+1)}{(x+4)^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(x+4)^n}(x+4)}{\cancel{3^n}\cdot 3 \cdot (n+2)} \cdot \frac{\cancel{3^n}(n+1)}{\cancel{(x+4)^n}} \right| \to \lim_{n\to\infty} \left| \frac{n+1}{n+2} \cdot \frac{x+4}{3} \right|$

$\left| \frac{x+4}{3} \right| < 1$

$-1 < \frac{x+4}{3} < 1$

$-3 < x+4 < 3$

$-7 < x < -1$

Endpts:

$x = -7$: $\frac{(-3)^n}{3^n(n+1)} = \frac{(-1)^n \cdot \cancel{3^n}}{\cancel{3^n}(n+1)}$

conv by AST

$x = -1$: $\frac{\cancel{3^n}}{\cancel{3^n}(n+1)} = \frac{1}{n+1}$ $\frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{n+1} \cdot \frac{n}{1} = 1$ $\therefore$ div by LCT

IOC: $-7 \le x < 1$

b) $f'(x) = \dfrac{n(x+4)^{n-1}}{3^n(n+1)}$

Endpts:

$x=-7$: $\dfrac{n(-3)^{n-1}}{3^n(n+1)} = \dfrac{n(-3)^n(-3)^{-1}}{3^n(n+1)} = \dfrac{n(-1)^n\cancel{(3)^n}}{-3\cdot\cancel{3^n}(n+1)}$

$\dfrac{(-1)^n\cdot n}{-3(n+1)}$ div by AST

$x=-1$: $\dfrac{n(3)^{n-1}}{3^n(n+1)} = \dfrac{n(3)^n(3)^{-1}}{3^n(n+1)} = \dfrac{n\cdot\cancel{3^n}}{3\cdot\cancel{3^n}(n+1)}$

$\dfrac{n}{3(n+1)} = \dfrac{n}{3n+3}$ div by n^{th} term

IOC: $-7 < x < -1$

c) $\int f(x)dx = \dfrac{(x+4)^{n+1}}{3^n(n+1)(n+1)} = \dfrac{(x+4)^{n+1}}{3^n(n+1)^2}$

Endpts:

$x=-7$: $\dfrac{(-3)^{n+1}}{3^n(n+1)^2} = \dfrac{(-3)^n(-3)}{3^n(n+1)^2} = \dfrac{(-1)^n\cancel{(3)^n}\cdot 3}{\cancel{3^n}(n+1)^2}$

$\dfrac{(-1)^n\cdot 3}{(n+1)^2}$ conv by AST

$x=-1$: $\dfrac{3^{n+1}}{3^n(n+1)^2} = \dfrac{\cancel{3^n}\cdot 3}{\cancel{3^n}(n+1)^2} = \dfrac{3}{n^2+2n+1}$ $\qquad \dfrac{3}{n^2}$

big conv

$\hookrightarrow$ conv by DCT

IOC: $-7 \le x \le -1$

4 a) $\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{n^2}$ $\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-5)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n (x-5)^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^n}(-1)\cancel{(x-5)^n}(x-5)}{(n+1)^2} \cdot \frac{n^2}{\cancel{(-1)^n (x-5)^n}} \right|$

$\lim_{n\to\infty} \left| \frac{n^2}{(n+1)^2} \cdot -(x-5) \right|$

$-1 < -x+5 < 1$

$-6 < -x < -4$

$6 > x > 4$

Endpts:

$x=4$: $\frac{(-1)^n(-1)^n}{n^2} = \frac{(-1)^{2n}}{n^2} = \frac{1}{n^2}$ conv p-series

$x=6$: $\frac{(-1)^n(1)^n}{n^2} = \frac{(-1)^n}{n^2}$ Conv AST

IOC: $4 \le x \le 6$

b) $f'(x) = \frac{(-1)^n n (x-5)^{n-1}}{n^2} = \frac{(-1)^n (x-5)^{n-1}}{n}$

Endpts:

$x=4$: $\frac{(-1)^n(-1)^{n-1}}{n} = \frac{(-1)^n(-1)^n(-1)^{-1}}{n} = \frac{(-1)^{2n}}{-n}$

$\frac{1}{-n}$ div p-series

$x=6$: $\frac{(-1)^n (1)^{n-1}}{n} = \frac{(-1)^n \cdot 1}{n}$ Conv AST

IOC: $4 < x \le 6$

c) $\int f(x)\,dx = \frac{(-1)^n (x-5)^{n+1}}{(n+1)\cdot n^2}$

$x=4$: $\frac{(-1)^n(-1)^{n+1}}{n^2(n+1)} = \frac{-1}{n^2(n+1)}$ conv by integral

$x=6$: $\frac{(-1)^n(1)^{n+1}}{n^2(n+1)} = \frac{(-1)^n}{n^2(n+1)}$ AST conv