KEY

5.5 Bases other than e

Find f′ $f(t)=\frac{3^{2t}}{t}$ $f'(t)=\frac{3^{2t}(2t\ln 3-1)}{t^2}$	Find f′ $f(x)=\log_5 \frac{x\sqrt{x-1}}{2}$ $f'(x)=$ $\frac{1}{x\ln 5}+\frac{1}{2(x-1)\ln 5}$	Find the equation of the tangent line at (2, 1), $y=5^{x-2}$ $y-1=\ln 5(x-2)$
Use log differentiation to find y′ $y=(1+x)^{\frac{1}{x}}$ $y'=\left(\frac{1}{x+x^2}-\frac{\ln(1+x)}{x^2}\right)(1+x)^{1/x}$	Find the equation of the tangent line at (π/2, 1), $y=(\sin x)^{2x}$ $y=1$	$\int 5^{-x}\,dx=$ $\frac{-5^{-x}}{\ln 5}+c$
$\int (3-x)7^{(3-x)^2}\,dx=$ $\frac{-7^{(3-x)^2}}{2\ln 7}+c$	$\int_1^e (6^x-2^x)dx$ $\left(\frac{6^e}{\ln 6}-\frac{2^e}{\ln 2}\right)-\left(\frac{6}{\ln 6}-\frac{2}{\ln 2}\right)$	Find the area of the region $y=3^{\cos x}\sin x$ bounded by y=0, x = 0, x = π $\frac{3}{\ln 3}-\frac{1}{3\ln 3}$

5.5 Bases other than e

*1) $f(t) = \dfrac{3^{2t}}{t}$ Quotient

$f'(t) = \dfrac{t(\ln 3 \cdot 3^{2t} \cdot 2) - 3^{2t}}{t^2}$

2) $f(x) = \log_5\left(\dfrac{x\sqrt{x-1}}{2}\right)$

$f(x) = \log_5 x + \frac{1}{2}\log_5(x-1) - \log_5 2$

$f'(x) = \dfrac{1}{\ln 5 \cdot x} + \dfrac{1}{2\ln 5 \cdot (x-1)}$

$\boxed{f'(x) = \dfrac{1}{x\ln 5} + \dfrac{1}{2(x-1)\cdot \ln 5}}$

3) $y = 5^{x-2}$ $(2,1)$

$y' = 5^{x-2} \cdot 1 \cdot \ln 5$

$y'(2) = 5^0 \cdot 1 \cdot \ln 5$

$y'(2) = \ln 5$

$y - 1 = \ln 5(x-2)$

4) $y = (1+x)^{1/x}$

$\ln y = \frac{1}{x} \cdot \ln(1+x)$

$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{x}\left(\frac{1}{1+x}\right) + \ln(1+x)\left(-\frac{1}{x^2}\right)\right)$

$\boxed{\dfrac{dy}{dx} = \left(\dfrac{1}{x+x^2} - \dfrac{\ln(1+x)}{x^2}\right)(1+x)^{1/x}}$

5) $y = (\sin x)^{2x} \quad \left(\frac{\pi}{2}, 1\right)$

$y' = (\sin x)^{2x} \cdot 2 \cdot \ln(\sin x)$

$y'\left(\frac{\pi}{2}\right) = \left(\sin \frac{\pi}{2}\right)^{2\left(\frac{\pi}{2}\right)} \cdot 2 \cdot \ln\left(\sin \frac{\pi}{2}\right)$

$y' = (1)^{\pi} \cdot 2 \cdot \underbrace{\cancel{\ln 1}}_{0}$

$y' = 0 \qquad \boxed{y = 1}$

6) * $\int 5^{-x}\,dx = \frac{-1}{\ln 5} \cdot 5^{-x} + C$

$u = -x$
$du = -dx$
$-du = dx$

$\boxed{-\frac{5^{-x}}{\ln 5} + C}$

7) * $\int (3-x) \cdot 7^{(3-x)^2}\,dx$

$u = (3-x)^2$
$du = -2(3-x)\,dx$
$-\frac{1}{2}\,du = (3-x)\,dx$

$-\frac{1}{2}\int 7^u\,du$

$= -\frac{1}{2} \cdot \frac{1}{\ln 7} \cdot 7^u + C = \boxed{\frac{-7^{(3-x)^2}}{2\ln 7} + C}$

8) $\int_1^e (6^x - 2^x)\,dx = \frac{1}{\ln 6} \cdot 6^x - \frac{1}{\ln 2} \cdot 2^x \Big|_1^e$

$\boxed{\left(\frac{1}{\ln 6} \cdot 6^e - \frac{1}{\ln 2} \cdot 2^e\right) - \left(\frac{1}{\ln 6} \cdot 6 - \frac{1}{\ln 2} \cdot 2\right)}$

9) $y = 3^{\cos x} \cdot \sin x \qquad \int_0^{\pi} 3^{\cos x} \cdot \sin x\,dx$

$u = \cos x$
$du = -\sin x\,dx$
$u(0) = 1$
$u(\pi) = -1$

$-\int_1^{-1} 3^u\,du = \int_{-1}^{1} 3^u\,du = \frac{1}{\ln 3} \cdot 3^u \Big|_{-1}^{1}$

$\boxed{\frac{3}{\ln 3} - \frac{1}{3\ln 3}}$