Day 1 Wrap Up:

① $f(x) = \frac{4\sqrt{2}}{3} x^{3/2}, \ 0 \le x \le 1$

$f'(x) = \frac{3}{2} \cdot \frac{4\sqrt{2}}{3} x^{1/2} = 2\sqrt{2} x^{1/2}$

$S = \int_0^1 \sqrt{1+(2\sqrt{2}x^{1/2})^2}\,dx = \int_0^1 \sqrt{1+8x}\,dx = \frac{13}{6} \approx 2.167$

② $f(x) = \frac{x^3}{6} + \frac{1}{2x} = \frac{1}{6}x^3 + \frac{1}{2}x^{-1} \quad [\frac{1}{2}, 2]$

$f'(x) = \frac{1}{2}x^2 - \frac{1}{2}x^{-2} = \frac{x^2}{2} - \frac{1}{2x}$

$S = \int_{1/2}^{2} \sqrt{1+\left(\frac{x^2}{2} - \frac{1}{2x}\right)^2}\,dx = 2.0625 \text{ or } \frac{33}{16}$

③ $(y-1)^3 = x^2 \quad [0,8]$

$y-1 = x^{2/3}$

$y = x^{2/3}+1$

$y' = \frac{2}{3}x^{-1/3} \qquad S = \int_0^8 \sqrt{1+(\frac{2}{3}x^{-1/3})^2}\,dx \approx 9.073$

④ $x = t^2-4$

$y = \frac{t}{2} \qquad 2y = t$

$x = (2y)^2-4$

$x = 4y^2-4$

⑤ $x = 3\cos\theta$

$y = 4\sin\theta$

$\left(\frac{x}{3} = \cos\theta\right)^2 \qquad \frac{x^2}{9} = \cos^2\theta$

$\left(\frac{y}{4} = \sin\theta\right)^2 \qquad \frac{y^2}{16} = \sin^2\theta$

$\cos^2\theta + \sin^2\theta = \frac{x^2}{9} + \frac{y^2}{16}$

ellipse! $\qquad 1 = \frac{x^2}{9} + \frac{y^2}{16}$

⑥ $x = \theta - \sin\theta$

$y = 1 - \cos\theta$

$$y' = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin\theta}{1-\cos\theta}$$

$$y'' = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\frac{dx}{d\theta}} = \frac{\frac{(1-\cos\theta)\cos\theta - \sin\theta(\sin\theta)}{(1-\cos\theta)^2}}{(1-\cos\theta)}$$

$$= \frac{\cos\theta - \cos^2\theta - \sin^2\theta}{(1-\cos\theta)^3} = \frac{\cos\theta - (\cos^2\theta + \sin^2\theta)}{(1-\cos\theta)^3}$$

$$= \frac{\cos\theta - 1}{(1-\cos\theta)^3} = \frac{-1(1-\cos\theta)}{(1-\cos\theta)^3} = \frac{-1}{(1-\cos\theta)^2}$$

⑦ $x = \sqrt{t+6} = (t+6)^{1/2}$

$y = \sqrt{6t} = (6t)^{1/2}$

$$y' = \frac{\frac{1}{2}(6t)^{-1/2}\cdot 6}{\frac{1}{2}(t+6)^{-1/2}} = \frac{\frac{3}{\sqrt{6t}}}{\frac{1}{2\sqrt{t+6}}} = \frac{3}{\sqrt{6t}}\cdot\frac{2\sqrt{t+6}}{1} = \frac{6\sqrt{t+6}}{\sqrt{6t}}$$

$$y'' = \frac{\frac{\sqrt{6t}\cdot\frac{6}{2\sqrt{t+6}} - 6\sqrt{t+6}\cdot\frac{1}{2\sqrt{6t}}\cdot 6}{(\sqrt{6t})^2}}{\frac{1}{2\sqrt{t+6}}}$$

$$= \frac{\frac{3\sqrt{6t}}{\sqrt{t+6}} \cdot \frac{-18\sqrt{t+6}}{\sqrt{6t}}}{6t} \cdot 2\sqrt{t+6}$$

$x = t^{-1}$ $\quad$ $t = -1$

$y = t$ $\quad$ $x = -1$

$y = -1$

$$\frac{dy}{dx} = \frac{1}{-\frac{1}{t^2}} = -t^2 \Big|_{t=-1} = -1$$

$$y+1 = -1(x+1)$$

9) $x = t^2 - t + 1$

$y = t^3 - 3t$

$$\frac{dy}{dx} = \frac{3t^2-3}{2t-1}$$

H: $3t^2 - 3 = 0$

$t^2 = 1$

$t = \pm 1$

$t = 1$ $(1, -2)$

$t = -1$ $(3, 2)$

$y = 2$

$y = -2$

V: $2t - 1 = 0$

$t = \frac{1}{2}$ $\left(\frac{3}{4}, \frac{-11}{8}\right)$

$x = \frac{3}{4}$

10) $$\int_0^{\pi} \sqrt{(e^t(-\sin t + \cos t))^2 + (e^t(\sin t + \cos t))^2}\, dt$$

≈ 31.312

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