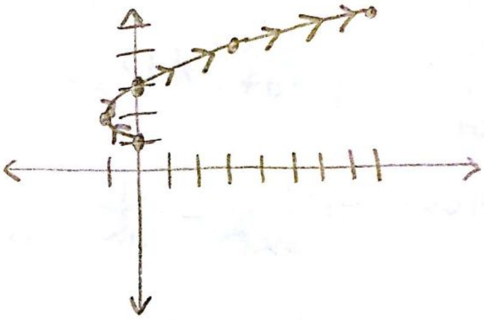
(10.2)

Parametric Warm-Up:

1. Graph $x = t^2 - 2t$ $\quad 0 \le t \le 4$
   $y = t + 1$

t	x	y
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5



2. Eliminate the parameter.

$t = y - 1$

$x = (y-1)^2 - 2(y-1)$

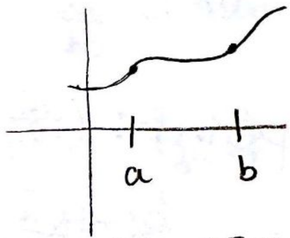
$x = y^2 - 2y + 1 - 2y + 2$

$x = y^2 - 4y + 3$

7.4 Arc Length

Arc Length (length of a curve):

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}$$



1. Length of curve $y = \ln(\cos x)$ from $[0, \frac{\pi}{3}]$

$$S = \int_0^{\pi/3} \sqrt{1 + (-\tan x)^2}\, dx \approx 1.317$$

$$y' = \frac{1}{\cos x} \cdot -\sin x = -\tan x$$

2. $x = y\sin y + \cos y$ from $y = 0$ to $y = \pi$

$$S = \int_0^{\pi} \sqrt{1 + (y\cos y)^2}\, dy \approx 4.848$$

$x' = y\cos y + \sin y - \sin y$

$x' = y\cos y$

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10.3 Parametric Equations and Calculus:

$x = f(t)$
$y = g(t)$

Slope $= \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$, $\dfrac{dx}{dt} \neq 0$

2^{nd} Derivative: $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)$

$$= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

1. $x = \sin t$, $y = \cos t$
   - $\frac{dy}{dx}$ when $t = \frac{\pi}{4}$
   - Write tan line when $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin t}{\cos t} = -\tan t \Big|_{t=\frac{\pi}{4}} = -1$$

point: $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ slope $= -1$

$$y - \frac{\sqrt{2}}{2} = -1\left(x - \frac{\sqrt{2}}{2}\right)$$

$x = \sqrt{t}$ $\quad t \geq 0$

$y = \frac{1}{4}(t^2 - 4)$

$\hookrightarrow \frac{1}{4}t^2 - 1$

- Find slope at (2, 3).
- Determine concavity at (2, 3).

$x = 2$
$2 = \sqrt{t}$
$t = 4$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{2}t}{\frac{1}{2}t^{-1/2}} = t^{3/2}\Big|_{t=4} = 8$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{3}{2}t^{1/2}}{\frac{1}{2}t^{-1/2}} = 3t\Big|_{t=4} = 12$$

$\therefore$ CC$\uparrow$

③ $x = \cos t$

$y = 3\sin t$

- Find slope and concavity at $t = 0$

$$\frac{dy}{dx} = \frac{3\cos t}{-\sin t} = -3\cot t\Big|_{t=0} \rightarrow \text{undefined}$$

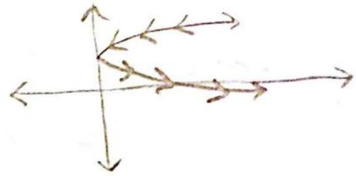
(vertical tan line)

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{3\csc^2 t}{-\sin t} = -3\csc^3 t\Big|_{t=0}$$

undefined
(possible POI)
neither CC$\uparrow$ nor CC$\downarrow$

$t = \pm\sqrt{x}$

$y = 2 \pm \sqrt{x}$



④ $x = t^2$

$y = 2 - t$

a) Determine concavity.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-1}{2t} = \frac{-1}{2}t^{-1}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{1}{2}t^{-2}}{2t} = \frac{1}{4t^3}$$

$= 0$ or undefined

undefined at $t=0$

$\frac{d^2y}{dx^2}$: $-$ | $+$

t: 0

cc $\downarrow$: $(-\infty, 0)$

cc $\uparrow$: $(0, \infty)$

Arc (Curve) Length in Parametric:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$$

① $x = 5\cos t - \cos 5t$

$y = 5\sin t - \sin 5t$

Length of curve over $[0, \pi/2]$

$$S = \int_0^{\pi/2} \sqrt{(-5\sin t + 5\sin 5t)^2 + (5\cos t - 5\cos 5t)^2}\, dt$$

$= 10$