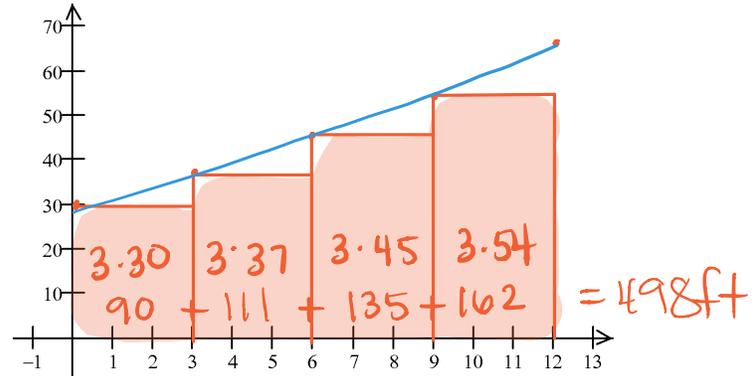


## Area under a Curve

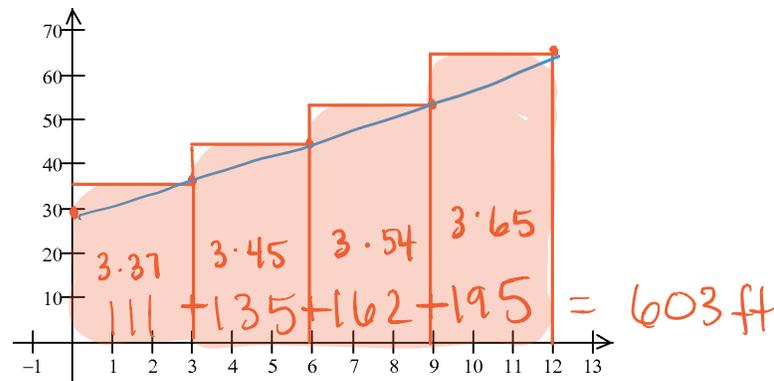
A car is traveling so that its speed is never decreasing during a 12-second interval. The speed at various moments in time is listed in the table below. For each part, consider how we could make a more accurate approximation.

Time (sec)	0	3	6	9	12
Speed (ft/sec)	30	37	45	54	65

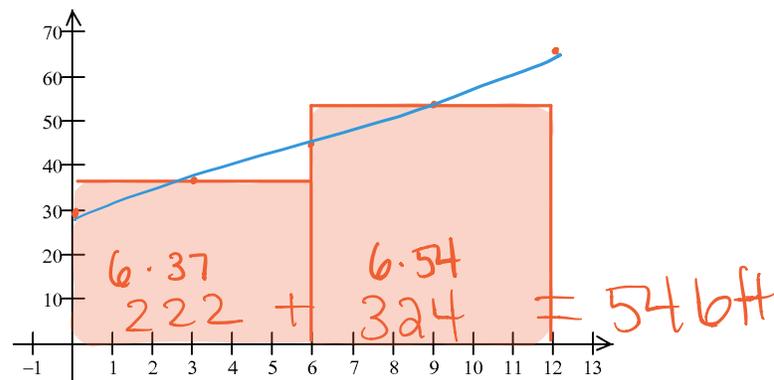
(a) Sketch a possible graph for this function, and draw **four** rectangles at the heights of the **left** endpoints. Then estimate the distance traveled by the car during the 12 seconds by finding the areas of the four rectangles and adding them together. This is called a **left Riemann sum**, named after the mathematician Georg Friedrich Riemann.



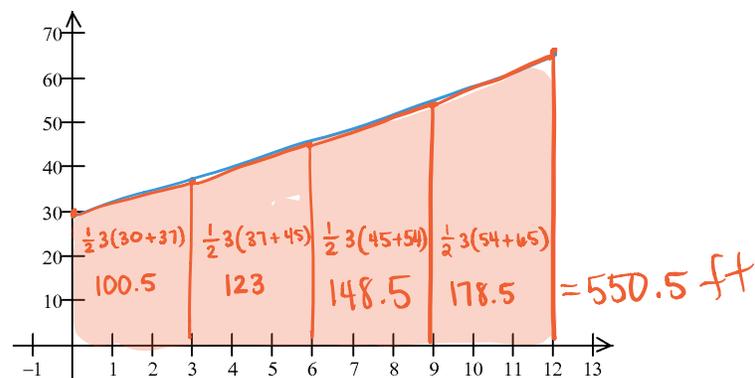
(b) Sketch a possible graph for this function, and draw **four** rectangles at the heights of the **right** endpoints. Then estimate the distance traveled by the car during the 12 seconds by finding the areas of the four rectangles and adding them together. This is called a **right Riemann sum**.



(c) Sketch a possible graph for this function, and draw **two** rectangles at the heights of the **midpoints**. Then estimate the distance traveled by the car during the 12 seconds by finding the areas of the two rectangles and adding them together. This is called a **midpoint Riemann sum**.



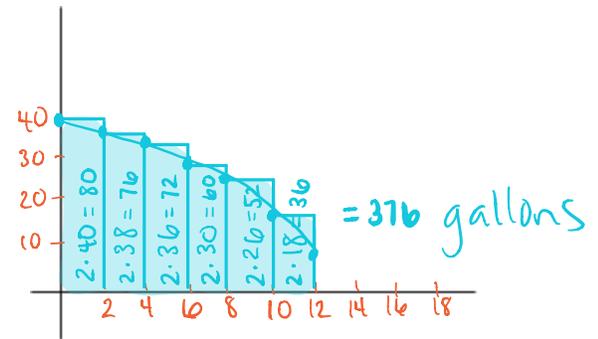
(d) Sketch a possible graph for this function, and draw **four** trapezoids with bases the lengths of the left and right **endpoints**. Then estimate the distance traveled by the car during the 12 seconds by finding the areas of the four trapezoids and adding them together. You've just approximated the area using the **Trapezoidal Rule**.



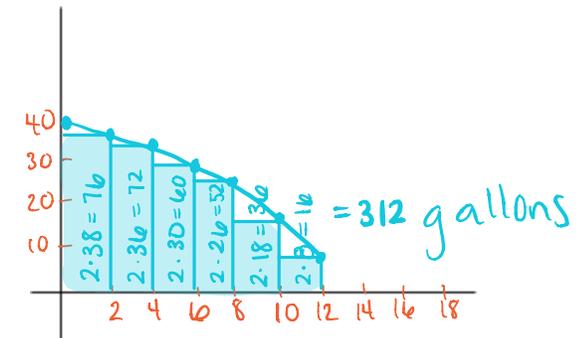
Oil is leaking out of a tank. The rate of flow is measured every two hours for a 12-hour period, and the data is listed in the table below.

Time (hr)	0	2	4	6	8	10	12
Rate (gal/hr)	40	38	36	30	26	18	8

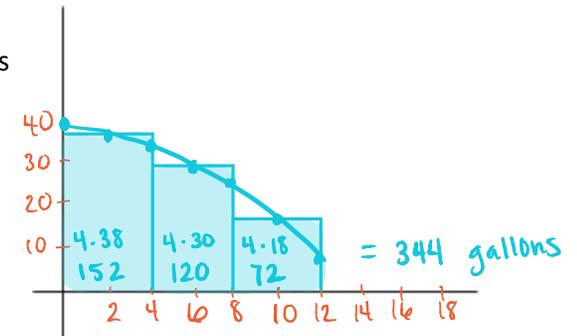
- (a) Sketch a possible graph for this function, and draw **six** rectangles at the heights of the **left** endpoints. Then estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding the areas of the six rectangles and adding them together. This is called a **left Riemann sum**.



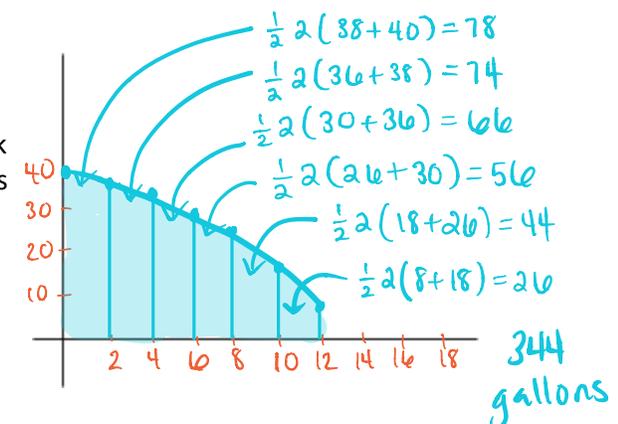
- (b) Sketch a possible graph for this function, and draw **six** rectangles at the heights of the **right** endpoints. Then estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding the areas of the six rectangles and adding them together. This is called a **right Riemann sum**.



- (c) Sketch a possible graph for this function, and draw **three** rectangles at the heights of the **midpoints**. Then estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding the areas of the three rectangles and adding them together. This is called a **midpoint Riemann sum**.



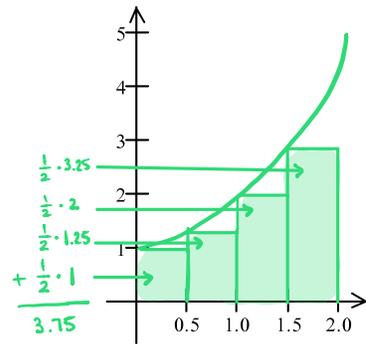
- (d) Sketch a possible graph for this function, and draw **six trapezoids** with bases at the left and right endpoints of the intervals. Then estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding the areas of the three rectangles and adding them together. This is an approximation using the **Trapezoidal Rule**.



Given the function  $y = x^2 + 1$ , estimate the area bounded by the graph of the curve and the x-axis from  $x = 0$  to  $x = 2$  by using:

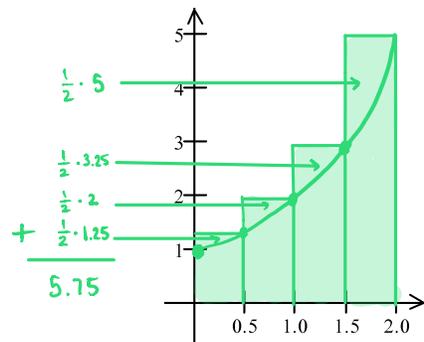
(a) a left Riemann sum with  $n = 4$  equal subintervals.

Draw the rectangles that you use.



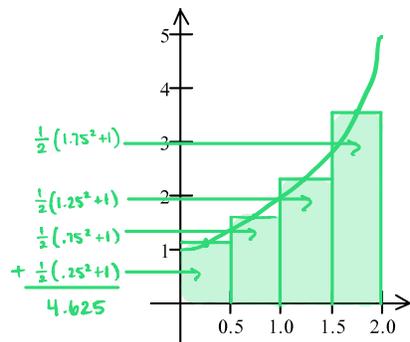
(b) a right Riemann sum with  $n = 4$  equal subintervals

Draw the rectangles that you use.



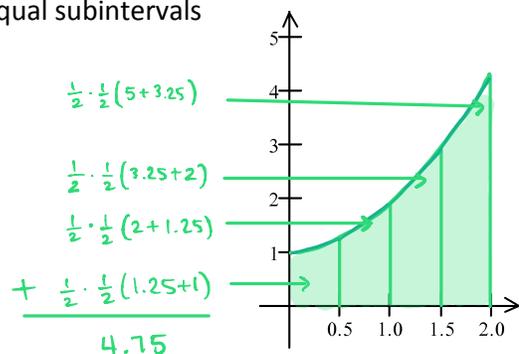
(c) a midpoint Riemann sum with  $n = 4$  equal subintervals

Draw the rectangles that you use.



(c) a Trapezoidal Rule approximation with  $n = 4$  equal subintervals

Draw the trapezoids that you use.



How could we get a more accurate estimate of the area bounded by the curve and the x-axis?

smaller rectangles / trapezoids