KEY

Day 7 – Curve Sketching

Objective: Use derivatives to sketch a graph

$y = 4x^3 - 3x - 1$ | $y' = 12x^2 - 3$ | $y'' = 24x$

Zeros:

$1 \,|\; 4 \quad 0 \quad -3 \quad -1$
$\quad\;\; 4 \quad 4 \quad 1$
$\quad\;\; 4 \quad 4 \quad 1 \quad 0$

$-\frac{1}{2} \,|\; 4 \quad 0 \quad -3 \quad -1$
$\quad\;\; -2 \quad -3 \quad -1$
$\quad\;\; 4 \quad -2 \quad -2 \quad 0$

Critical Points:

$12x^2 - 3 = 0$

$12x^2 = 3$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

Possible POI:

$24x = 0$

$x = 0$

Sign Chart for f′

$f'(x)$: $+$ | $-\frac{1}{2}$ | $-$ | $\frac{1}{2}$ | $+$

x

Sign chart for f″

$f''(x)$: $-$ | 0 | $+$

x

Maximums (ordered pair) $\left(-\frac{1}{2}, 0\right)$

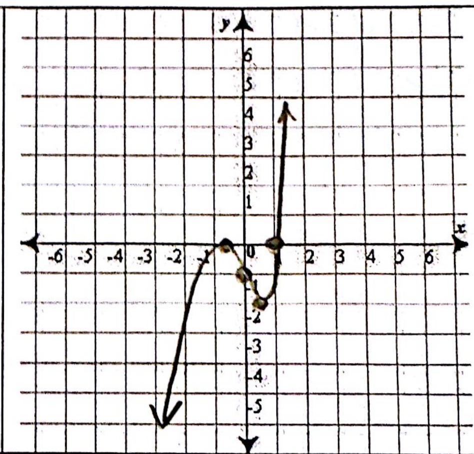
Minimums (ordered pair) $\left(\frac{1}{2}, -2\right)$

POI (ordered pair) $(0, -1)$

Intervals of increasing $\left(-\infty, -\frac{1}{2}\right)$, $\left(\frac{1}{2}, \infty\right)$

Intervals of decreasing $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Intervals of concavity CC↑ $(0, \infty)$; CC↓ $(-\infty, 0)$



$9x^2 - x^4 - 36 + 4x^2 = \frac{1}{4}(-x^4 + 13x^2 - 36) = -\frac{1}{4}x^4 + \frac{13}{4}x^2 - 9$

$y = \frac{1}{4}(x^2 - 4)(9 - x^2)$ | $y' = -x^3 + \frac{13}{2}x$ | $y'' = -3x^2 + \frac{13}{2} = 0$

Zeros:

$x = 2, -2, 3, -3$

Critical Points:

$-x\left(x^2 - \frac{13}{2}\right) = 0$

$x = 0 \quad x^2 = \frac{13}{2}$

$x = \pm\sqrt{\frac{13}{2}}$

Possible POI:

$-3x^2 = -\frac{13}{2}$

$3x^2 = \frac{13}{2}$

$x^2 = \frac{13}{6}$

$x = \pm\sqrt{\frac{13}{6}}$

Sign Chart for f′

$f'(x)$: $+$ | $-\sqrt{\frac{13}{2}}$ | $-$ | 0 | $+$ | $\sqrt{\frac{13}{2}}$ | $-$

x

Sign chart for f″

$f''(x)$: $-$ | $-\sqrt{\frac{13}{6}}$ | $+$ | $\sqrt{\frac{13}{6}}$ | $-$

x

Maximums (ordered pair) $\left(-\sqrt{\frac{13}{2}}, \frac{25}{16}\right)$, $\left(\sqrt{\frac{13}{2}}, \frac{25}{16}\right)$

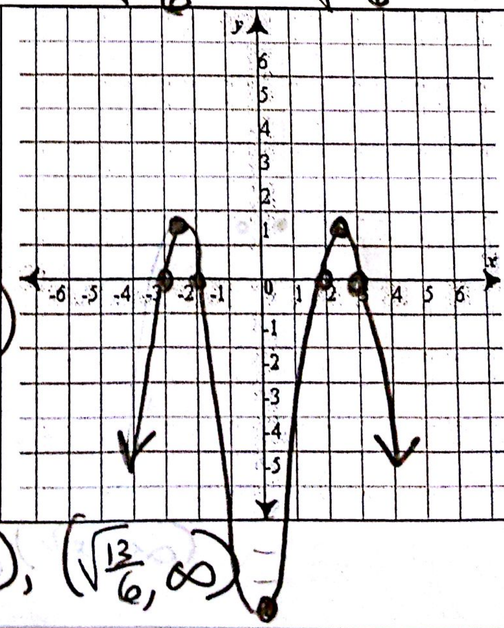
Minimums (ordered pair) $(0, -9)$

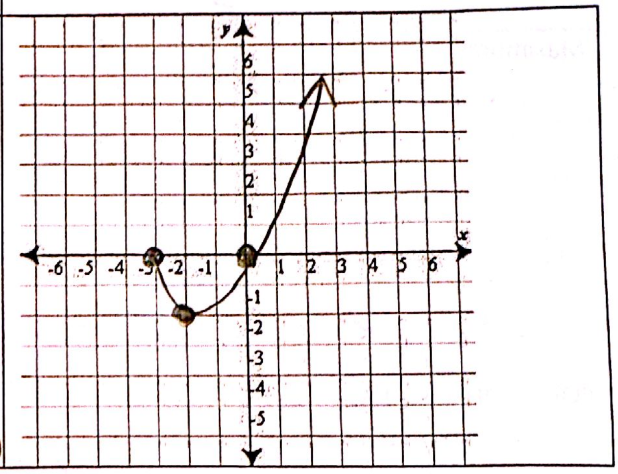
POI (ordered pair) $\left(-\sqrt{\frac{13}{6}}, \frac{41}{24}\right)$, $\left(\sqrt{\frac{13}{6}}, \frac{41}{24}\right)$

Intervals of increasing $\left(-\infty, -\sqrt{\frac{13}{2}}\right)$, $\left(0, \sqrt{\frac{13}{2}}\right)$

Intervals of decreasing $\left(-\sqrt{\frac{13}{2}}, 0\right)$, $\left(\sqrt{\frac{13}{2}}, \infty\right)$

Intervals of concavity CC↑ $\left(-\sqrt{\frac{13}{6}}, \sqrt{\frac{13}{6}}\right)$; CC↓ $\left(-\infty, -\sqrt{\frac{13}{6}}\right)$, $\left(\sqrt{\frac{13}{6}}, \infty\right)$



$y = x\sqrt{x+3}$ $x(x+3)^{1/2}$	$y' =$ $\frac{3x+6}{2\sqrt{x+3}}$	$y'' =$ $\frac{3(x+4)}{4(x+3)(\sqrt{x+3})}$
Zeros: $x=0, -3$	Critical Points: $x=-2$ $x=-3$	Possible POI: $x=-4$
Sign Chart for f' $f'(x)$ DNE | $-$ | $+$ x -3 -2	Sign chart for f'' $f''(x)$ | | $+$ x DNE -4 -3	
Maximums (ordered pair) none Minimums (ordered pair) $(-2,-2)$ POI (ordered pair) none	Intervals of increasing $(-2,\infty)$ Intervals of decreasing $(-3,-2)$ Intervals of concavity CC↑$(-3,\infty)$ CC↓ none	

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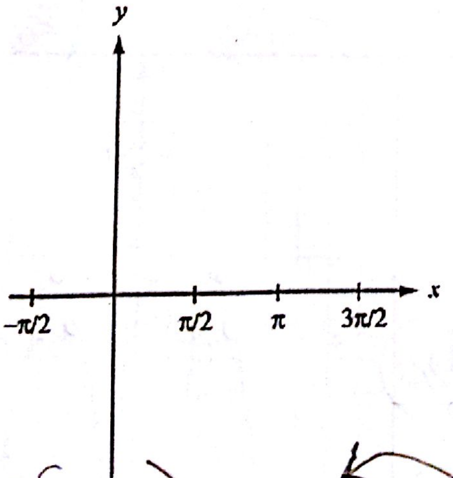
Given the function defined by $y = x + \sin x$ for all x such that $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$.

(a) Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.

(b) Find the coordinates of all points of inflection on the given interval. Justify your answers.

(c) On the axes provided, sketch the graph of the function.



$$f'(x) = x\left(\tfrac{1}{2}(x+3)^{-1/2}\right) + \sqrt{x+3}$$

$$\frac{x}{2\sqrt{x+3}} + \sqrt{x+3} \cdot \frac{2\sqrt{x+3}}{2\sqrt{x+3}}$$

$$\frac{x}{2\sqrt{x+3}} + \frac{2(x+3)}{2\sqrt{x+3}}$$

$$\frac{3x+6}{2\sqrt{x+3}} \qquad 2(x+3)^{1/2}$$

$$f''(x) = \frac{(2\sqrt{x+3})(3) - (3x+6)(x+3)^{-1/2}}{(2\sqrt{x+3})^2}$$

$$= \frac{6\sqrt{x+3} - \frac{3x+6}{\sqrt{x+3}}}{4(x+3)} \cdot \frac{\sqrt{x+3}}{\sqrt{x+3}}$$

$$6x+18-3x-6$$

$$\frac{6(x+3) - (3x+6)}{4(x+3)\sqrt{x+3}} = \frac{3x+12}{4(x+3)\sqrt{x+3}}$$

$$\frac{3(x+4)}{4(x+3)(\sqrt{x+3})}$$

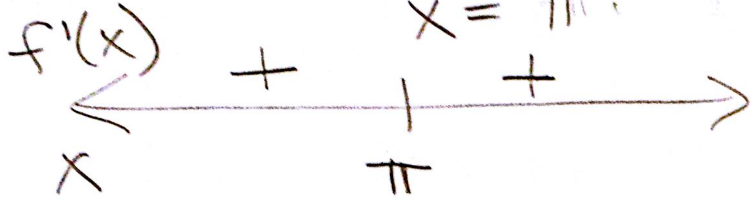
R:

$y = x + \sin x \; , \; -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

a) $y' = 1 + \cos x = 0$

$\cos x = -1$

$x = \pi$



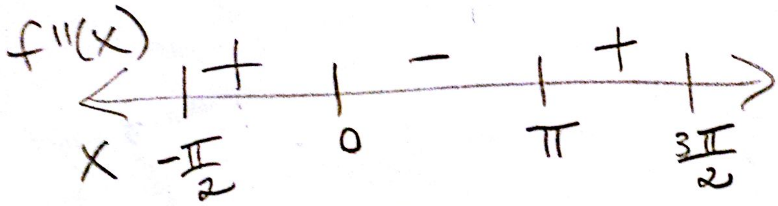
x	y
$-\frac{\pi}{2}$	$-\frac{\pi}{2} + \sin(-\frac{\pi}{2}) = -\frac{\pi}{2} - 1$
$\frac{3\pi}{2}$	$\frac{3\pi}{2} + \sin(\frac{3\pi}{2}) = \frac{3\pi}{2} - 1$

Abs min: $(-\frac{\pi}{2}, -\frac{\pi}{2} - 1)$

Abs max: $(\frac{3\pi}{2}, \frac{3\pi}{2} - 1)$

b) $y'' = -\sin x = 0$

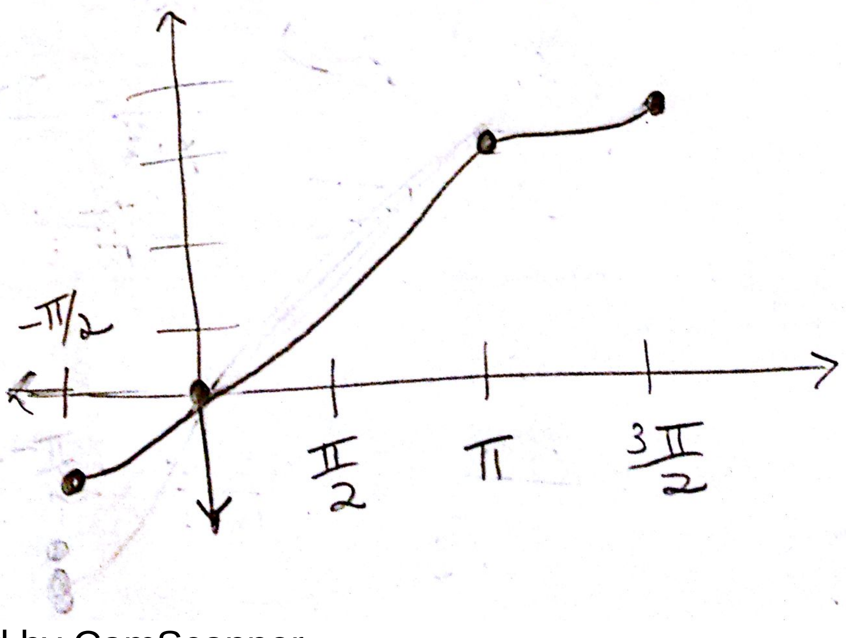
$x = 0, \pi$



POI: $(0, 0)$

(π, π)

c)



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