Name: KEY

Date:

CURVE SKETCHING
Section 3.6

Complete the following table. Sketch the functions based on the information for the table.

	(1)	(2)	(3)	(4)
Function	$y = 3x^4 + 4x^3$	$Y = \frac{x^2 - 2x + 4}{x - 2}$	$y = 3x^{\frac{2}{3}} - 2x$	$Y = \frac{\cos(x)}{1 + \sin(x)}$ use $(0, 2\pi)$
First Derivative	$f'(x) = 12x^3 + 12x^2$ $12x^2(x+1)$	$y' = \frac{x^2 - 4x}{(x-2)^2}$	$y' = \frac{2}{3\sqrt[3]{x}} - 2$ $2x^{-1/3} - 2$	$y' = \frac{-1}{1 + \sin x}$
Second Derivative	$f''(x) =$ $36x^2 + 24x$ $= 12x(3x + 2)$	$y'' = \frac{8}{(x-2)^3}$	$y'' = \frac{-2}{3\sqrt[3]{x^4}}$ $= \frac{2}{3}x^{-4/3}$	$y'' = \frac{\cos x}{(1 + \sin x)^2}$
x-intercept(s)/zeros	$(0,0)$ $(-4/3, 0)$	none	$(0,0)$ $(27/8, 0)$	$(\frac{\pi}{2}, 0)$
y-intercept	$(0,0)$	$(0,-2)$	$(0,0)$	$(0,1)$
Vertical Asymptotes	none	$x = 2$	none	$x = \frac{3\pi}{2}$
Horizontal Asymptotes	none	none	none	none

CURVE SKETCHING
Section 3.6

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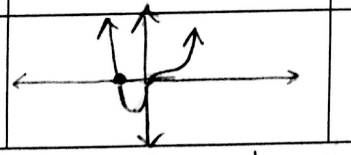
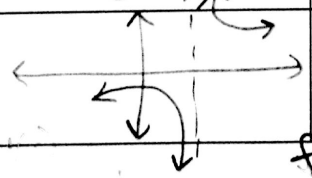
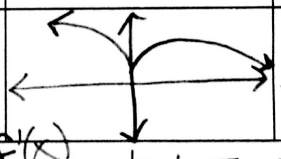
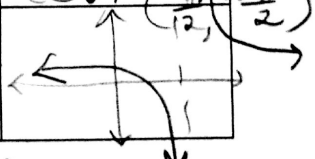
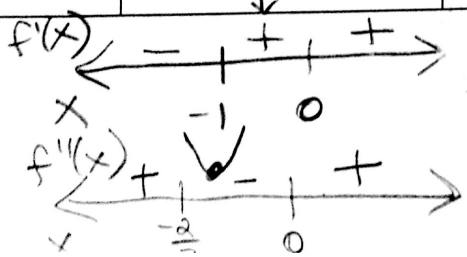
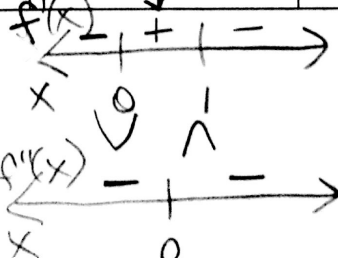
Critical numbers	$x = 0, -1$	$x = 0, 2, 4$	$x = 0, 1$	$x = \frac{3\pi}{2}$
Interval(s) increasing/Decreasing	Inc: $(-1, 0), (0, \infty)$ Dec: $(-\infty, -1)$	Inc: $(-\infty, 0)$, $(4, \infty)$ Dec: $(0, 2), (2, 4)$	Inc: $(0, 1)$ Dec: $(-\infty, 0)$, $(1, \infty)$	Inc: none Dec: $(0, \frac{3\pi}{2}), (\frac{3\pi}{2}, 2\pi)$
Extrema	Rel min: $(-1, -1)$	Rel max: $(0, -2)$ Rel min: $(4, 6)$	Rel max: $(1, 1)$ Rel min: $(0, 0)$	none
PossiblePoints of inflection	$x = -2/3$ $x = 0$	$x = 2$ Not a POI (VA)	$x = 0$	$x = \frac{\pi}{2}$ $(\frac{\pi}{2}, 0)$
Concavity UP/Down	CC↑ $(-\infty, -2/3), (0, \infty)$ CC↓ $(-\frac{2}{3}, 0)$	CC↑ $(2, \infty)$ CC↓ $(-\infty, 2)$	CC↓ $(-\infty, 0), (0, \infty)$	CC↑: $(0, \frac{\pi}{2}), (\frac{3\pi}{2}, 2\pi)$ CC↓: $(\frac{\pi}{2}, \frac{3\pi}{2})$
Sketch the function				





② $y = \dfrac{x^2 - 2x + 4}{x - 2}$

$y' = \dfrac{(x-2)(2x-2) - [(x^2-2x+4)(1)]}{(x-2)^2}$

$y' = \dfrac{2x^2 - 2x - 4x + 4 - x^2 + 2x - 4}{(x-2)^2}$

$y' = \dfrac{x^2 - 4x}{(x-2)^2} = \dfrac{x(x-4)}{(x-2)^2}$

$y'' = \dfrac{(x-2)^2(2x-4) - [(x^2-4x)(2(x-2))]}{(x-2)^4}$

$y'' = \dfrac{2(x-2)^2(x-2) - [2x(x-4)(x-2)]}{(x-2)^4}$

$y'' = \dfrac{2(x-2)[(x-2)^2 - [x(x-4)]]}{(x-2)^{4\,3}}$

$y'' = \dfrac{2(x^2 - 4x + 4 - x^2 + 4x)}{(x-2)^3}$

$y'' = \dfrac{8}{(x-2)^3}$

x-int:

$0 = x^2 - 2x + 4$

$x = \dfrac{2 \pm \sqrt{4 - 4(1)(4)}}{2}$

$x = \dfrac{2 \pm \sqrt{-12}}{2}$ imaginary solutions

y-int: $(0, -2)$

VA: $x = 2$

HA: none (oblique!)

crit #'s: $x = 0, 2, 4$

$f'(x)$: + | − | − | +

x: 0, 2, 4

Inc: $(-\infty, 0)$, $(4, \infty)$

Dec: $(0, 2)$, $(2, 4)$

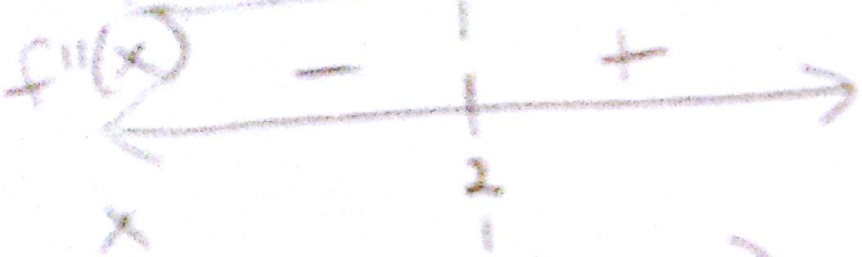
Rel max: $(0, -2)$

Rel min: $(4, 6)$

② cont'd...

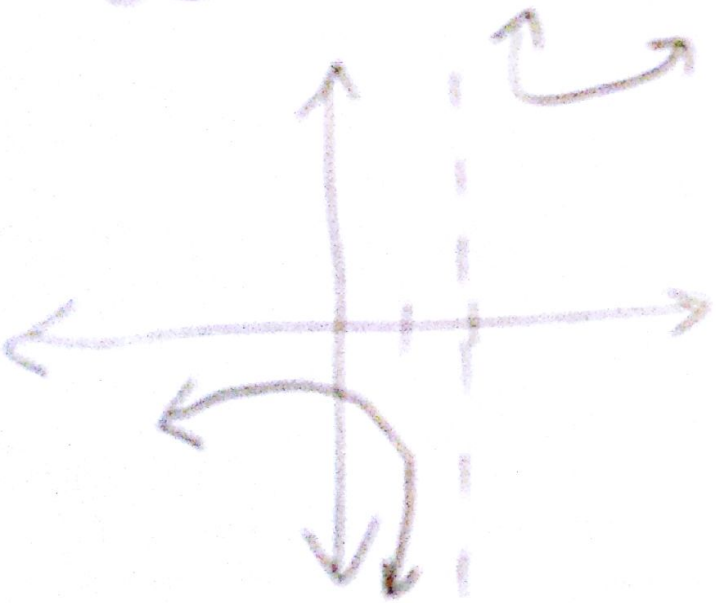
Possible PoI: $x = 2$

not a PoI (VA)



CC↑ : $(2, \infty)$

CC↓ : $(-\infty, 2)$



④ $y = \dfrac{\cos x}{1+\sin x}$

$$y' = \frac{(1+\sin x)(-\sin x) - (\cos x)(\cos x)}{(1+\sin x)^2}$$

$$-\sin x - \sin^2 x - \cos^2 x$$

$$-\sin x - \sin^2 x - (1 - \sin^2 x)$$

$$-\sin x - \cancel{\sin^2 x} - 1 + \cancel{\sin^2 x}$$

$$\frac{-\sin x - 1}{(1+\sin x)^2} = \frac{-(\cancel{\sin x + 1})}{(1+\sin x)^{\cancel{2}}}$$

$$\boxed{y' = \frac{-1}{1+\sin x}} = -(1+\sin x)^{-1}$$

$$y'' = (1+\sin x)^{-2} \cdot \cos x$$

$$\boxed{y'' = \frac{\cos x}{(1+\sin x)^2}}$$

y-int: $\dfrac{\cos 0}{1+\sin 0} =$

$\dfrac{1}{1+0} = 1$

y-int: $\boxed{(0, 1)}$

x-ints: $\cos x = 0$

$x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}$

$\boxed{\left(\frac{\pi}{2}, 0\right)}$

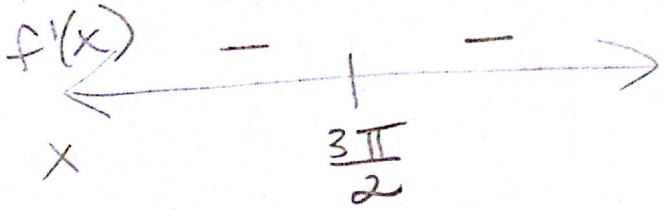
$\boxed{VA: x = \frac{3\pi}{2}}$ $1 + \sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

$\boxed{HA;\ none}$

crit #'s: $\frac{3\pi}{2}$



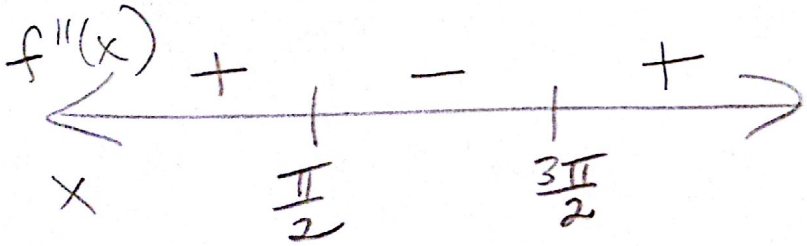
Dec: $(0, \frac{3\pi}{2}), (\frac{3\pi}{2}, 2\pi)$

Inc: none

Extrema: none

Possible POI: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

POI: $(\frac{\pi}{2}, 0)$



CC↑: $(0, \frac{\pi}{2}), (\frac{3\pi}{2}, 2\pi)$

CC↓: $(\frac{\pi}{2}, \frac{3\pi}{2})$