AP Calculus AB
Worksheet on Cross Sections
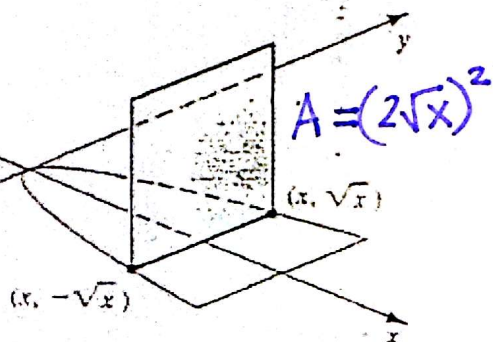
Name: __Key__

Date: _____

Exer. 1–8: Let R be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the volume of the solid that has R as its base if every cross section by a plane perpendicular to the x-axis has the given shape.
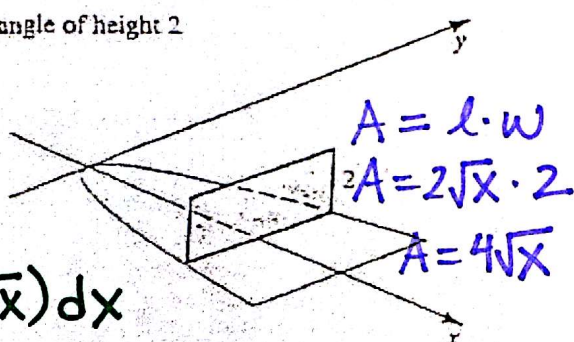
1 A square
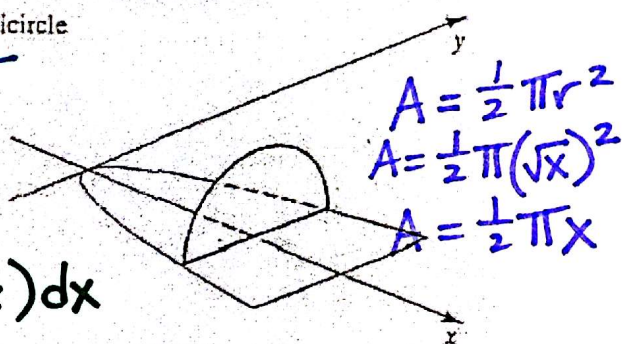
162

$$V = \int_0^9 (2\sqrt{x})^2 \, dx$$

$A = (2\sqrt{x})^2$

$(x, \sqrt{x})$

$(x, -\sqrt{x})$

2 A rectangle of height 2

72

$$V = \int_0^9 (4\sqrt{x}) \, dx$$

$A = l \cdot w$
$A = 2\sqrt{x} \cdot 2$
$A = 4\sqrt{x}$

3 A semicircle

$\dfrac{81\pi}{4}$

$$V = \int_0^9 \left(\tfrac{1}{2}\pi x\right) dx$$

$A = \tfrac{1}{2}\pi r^2$
$A = \tfrac{1}{2}\pi(\sqrt{x})^2$
$A = \tfrac{1}{2}\pi x$

4 A quartercircle

$\dfrac{81\pi}{2}$

$$V = \int_0^9 (\pi x) \, dx$$

$A = \tfrac{1}{4}\pi r^2$
$A = \tfrac{1}{4}\pi(2\sqrt{x})^2$
$A = \pi x$
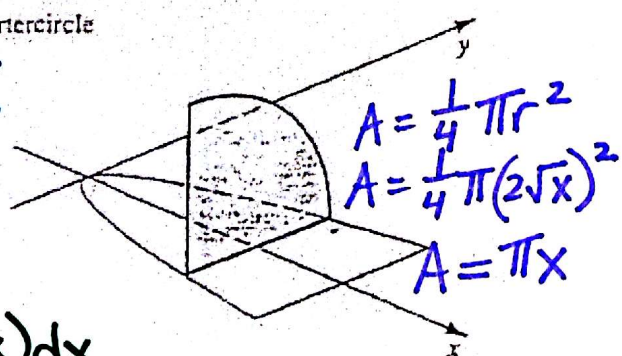
5 An equilateral triangle

$A = \dfrac{s^2\sqrt{3}}{4}$

$\dfrac{81\sqrt{3}}{2}$

$A = \dfrac{(2\sqrt{x})^2 \cdot \sqrt{3}}{4}$
$A = x\sqrt{3}$
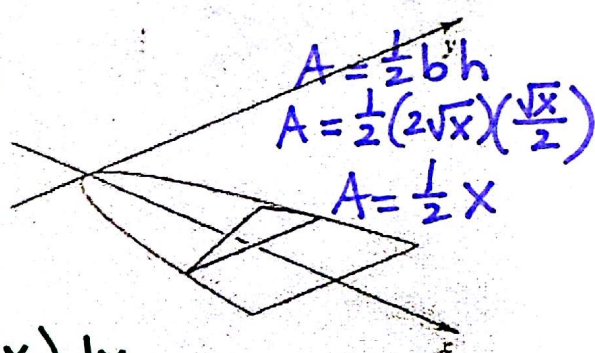
$$\int_0^9 (x\sqrt{3}) \, dx$$
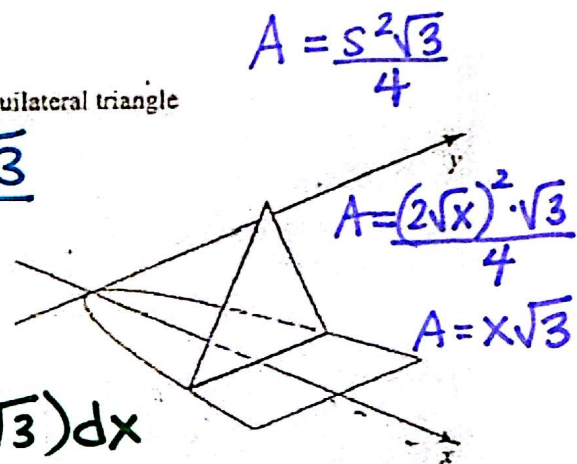
6 A triangle with height equal to $\tfrac{1}{4}$ the length of the base

$\dfrac{81}{4}$

$A = \tfrac{1}{2}bh$
$A = \tfrac{1}{2}(2\sqrt{x})\left(\dfrac{\sqrt{x}}{2}\right)$
$A = \tfrac{1}{2}x$

$$\int_0^9 \left(\tfrac{1}{2}x\right) dx$$

7 A trapezoid with lower base in the xy-plane, upper base equal to $\tfrac{1}{2}$ the length of the lower base, and height equal to $\tfrac{1}{4}$ the length of the lower base

$\dfrac{243}{8}$

$$\int_0^9 \left(\tfrac{3}{4}x\right) dx$$

$A = \tfrac{1}{2}h(b_1 + b_2)$
$A = \tfrac{1}{2}\left(\tfrac{1}{2}\sqrt{x}\right)\left(\sqrt{x} + 2\sqrt{x}\right)$
$A = \tfrac{3}{4}x$