AP Calculus AB
Cross Sections Worksheet

*K FRQ

Name Key

1. The base of a solid is bounded by $y = \cos(x)$, the x-axis, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Cross sections perpendicular to the x-axis are squares. Find the volume.

$S = \cos x \quad A = (\cos x)^2$ $\quad V = \int_{-\pi/2}^{\pi/2} (\cos x)^2 dx = 1.571$

2. The base of a solid is bounded by $y = 2 - x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the x-axis are isosceles right triangles with the right angle on the x-axis. (Legs perpendicular to the x-axis). Find the volume.

$A = \frac{1}{2}bh = \frac{1}{2}(2-x)^2$ $\quad V = \frac{1}{2}\int_0^2 (2-x)^2 dx = 1.333$

3. The base of a solid is bounded by the semi-circle $y = \sqrt{4 - x^2}$ and the x-axis. Cross sections that are perpendicular to the x-axis are squares. Find the volume.

$S = \sqrt{4-x^2} \quad A = (\sqrt{4-x^2})^2$ $\quad V = \int_{-2}^{2} (4-x^2) dx = 10.667$

4. The base of a solid is bounded by $y = \sqrt{16 - x^2}$ and the x-axis. Cross sections that are perpendicular to the y-axis are equilateral triangles. Find the volume.

$S = 2\sqrt{16-y^2} \quad A = \frac{(2\sqrt{16-y^2})^2 \cdot \sqrt{3}}{4}$ $\quad \sqrt{3}\int_0^4 (16-y^2) dy = 73.901$

5. The base of a solid is a circular region in the xy-plane bounded by the graph $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is an equilateral triangle with one side as the base.

$S = 2\sqrt{9-x^2} \quad A = \frac{(2\sqrt{9-x^2})^2 \cdot \sqrt{3}}{4}$ $\quad V = \sqrt{3}\int_{-3}^{3} (9-x^2) dx = 62.354$

6. The base of a solid is circular region in the xy-plane bounded by the graph of $x^2 + y^2 = 9$. Find the volume of the solid if every cross section by a plane normal to the x-axis is a square with one side as the base.

$S = 2\sqrt{9-x^2} \quad A = (2\sqrt{9-x^2})^2$ $\quad V = \int_{-3}^{3} (4(9-x^2)) dx = 144$

7. The base of a solid is bounded by $y = 2 - \frac{1}{2}x$, the x-axis, and the y-axis. Cross sections that are perpendicular to the y-axis are isosceles right triangles with the hypotenuse in the xy-plane. Find the volume.

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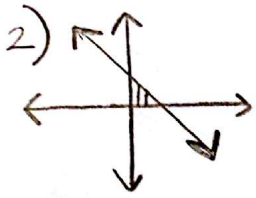
2.667

Answers

1. $\frac{\pi}{2}$
2. $\frac{4}{3}$
3. $\frac{32}{3}$
4. $\frac{128\sqrt{3}}{3}$
5. $36\sqrt{3}$
6. 144
7. $\frac{8}{3}$

2)



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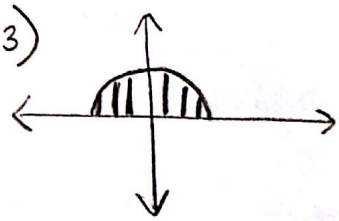

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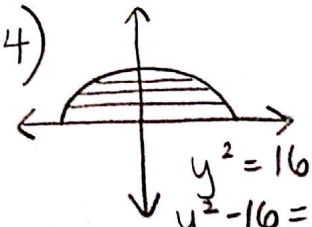
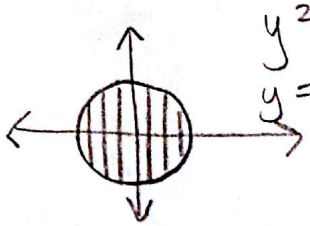


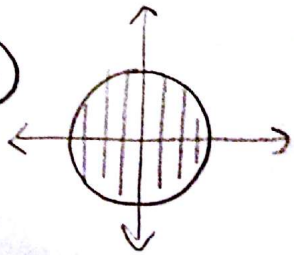
$y^2 = 16 - x^2$
$y^2 - 16 = -x^2$
$16 - y^2 = x^2$
$x = \pm\sqrt{16 - y^2}$

5)



$y^2 = 9 - x^2$
$y = \pm\sqrt{9 - x^2}$

6)



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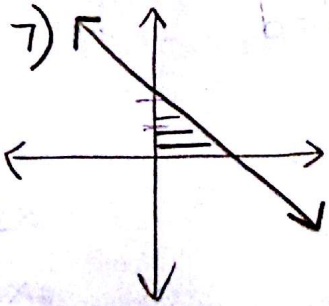

7)

$-2y+4$

$y = 2 - \frac{1}{2}x$

$y - 2 = -\frac{1}{2}x$

$-2y + 4 = x$

$a^2 + a^2 = (-2y+4)^2$

$\sqrt{2a^2} = \sqrt{(-2y+4)^2}$

$a\sqrt{2} = -2y+4$

$a = -\frac{2y+4}{\sqrt{2}}$

$A = \frac{1}{2}\left(\frac{(-2y+4)}{\sqrt{2}}\right)^2$

$V = \frac{1}{2}\int_0^2 \frac{(-2y+4)^2}{2}\,dy$

$V = \frac{1}{4}\int_0^2 (-2y+4)^2\,dy$