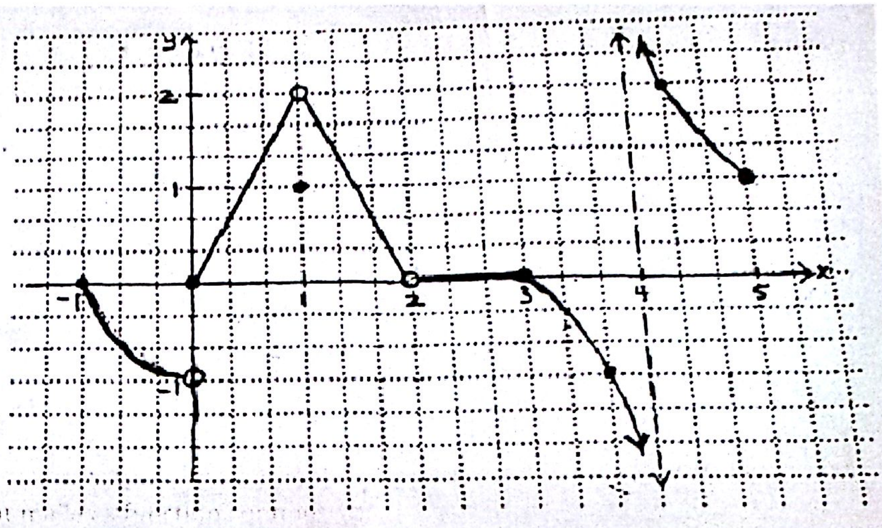
Sammit - key

What makes a function continuous?

The Meaning of Continuity:

Consider the function f defined below:

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \\ \dfrac{1}{2(x-4)}+0.5, & 3<x\le 5,\ x\ne 4 \end{cases}$$



1. Over the interval [-1, 5], where is the function f discontinuous?

at $x = 0, 1, 2, 4$

2. Complete the table for each value of x = c given.

X = c	$f(c)$	$\lim_{x\to c^-} f(x)$	$\lim_{x\to c^+} f(x)$	$\lim_{x\to c} f(x)$	Does $f(c)=\lim_{x\to c} f(x)$	Is $f(x)$ continuous at x = c?
0	0	-1	0	DNE	No	No
1	1	2	2	2	No	No
2	DNE	0	0	0	No	No
3	0	0	0	0	Yes	Yes
4	DNE	$-\infty$	∞	DNE	DNE N/A	No

Using the table above determine the three conditions that must exist in order for a function to be continuous:

a) The $\lim_{x\to c} f(x)$ exists

b) $f(c)$ is defined

c) $\lim_{x\to c} f(x)$ equals $f(c)$

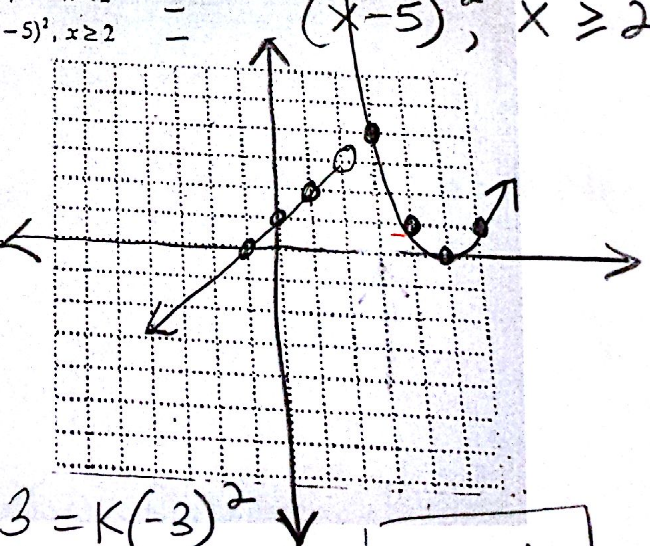
EXAMPLE: Let f be the function defined by $f(x)=\begin{cases} x+1, & x<2 \\ k(x-5)^2, & x\ge 2 \end{cases}$

$x+1;\ x<2$

$(x-5)^2;\ x\ge 2$

where k stands for a constant.

1. Graph this function for $k=1$.



2. Use the definition of continuity to explain formally what must be true in order for f to be continuous at $x=2$.

The limit has to exist

$\lim_{x\to 2} = f(2)$

3. Find the value of k that makes f continuous at x = 2. Then sketch f for this value of k.

$f(2)=9$

$\lim_{x\to 2} = \text{DNE}$

$x+1 = k(x-5)^2$

$2+1 = k(2-5)^2$

$3 = k(-3)^2$

$\frac{3}{9} = \frac{9k}{9}$

$\boxed{k=\frac{1}{3}}$