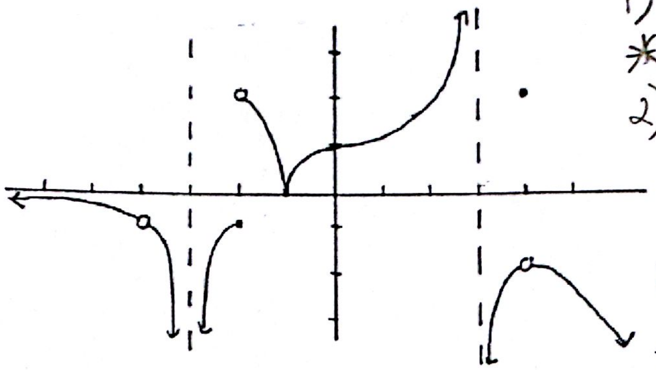
Do: 8, 9, 11–14

Use the graph below.

a) Find all x values where the graph is discontinuous.
b) For each discontinuous x value, name the type of discontinuity.
c) For each discontinuous x value, state why it fails the formal definition of continuity.



1) $x = -4$, hole
$*\ f(-4)$ DNE

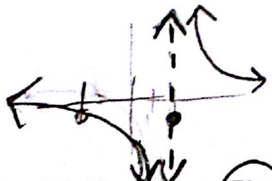
2) $x = -3$, asy
$*\ \lim\limits_{x \to -3}$ DNE

3) $x = -2$, break
$*\ \lim\limits_{x \to -2}$ DNE

4) $x = 3$, asy
$*\ \lim\limits_{x \to 3}$ DNE

5) $x = 4$, break
$\lim\limits_{x \to 4} \neq f(4)$

7. $f(x) = \begin{cases} \dfrac{x^2+5x+6}{x^2-4}, & if\ x \neq \pm 2 \\ -\dfrac{1}{4}, & if\ x = \pm 2 \end{cases}$



Where is $f(x)$ discontinuous? (A) Nowhere (B) $x = 2$ (C) $x = -2$ (D) $x = \pm 2$ (E) $\mathbb{R}$

$\dfrac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$

hole @ $x = -2$
VA @ $x = 2$

$\dfrac{-2+3}{-2-2} = \dfrac{1}{-4}$

Find the value of a and b so that $f(x)$ is continuous

8. $f(x) = \begin{cases} ax-1, & x < -1 \\ -x^2+1, & -1 \leq x < 2 \\ \frac{1}{2}x + b, & x \geq 2 \end{cases}$

9. $f(x) = \begin{cases} 2x+a, & x \leq -1 \\ x^2+1, & -1 < x \leq 2 \\ bx-1, & x > 2 \end{cases}$

$ax - 1 = -x^2 + 1$
$-1a - 1 = -(-1)^2 + 1$
$-1a - 1 = -1 + 1$
$-1a - 1 = 0$
$-1a = 1$
$\boxed{a = -1}$

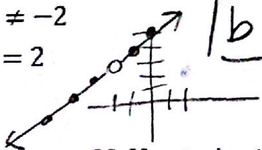
$-x^2 + 1 = \frac{1}{2}x + b$
$-(2)^2 + 1 = \frac{1}{2}(2) + b$
$-4 + 1 = 1 + b$
$-3 = 1 + b$
$\boxed{b = -4}$

$2x + a = x^2 + 1$
$2(-1) + a = (-1)^2 + 1$
$-2 + a = 1 + 1$
$-2 + a = 2$
$\boxed{a = 4}$

$x^2 + 1 = bx - 1$
$2^2 + 1 = b(2) - 1$
$5 = 2b - 1$
$6 = 2b$
$\boxed{b = 3}$

10. $f(x) = \begin{cases} \frac{x^2+7x+10}{x+2} & if\ x \neq -2 \\ b & if\ x = 2 \end{cases}$

$\dfrac{(x+5)\cancel{(x+2)}}{\cancel{(x+2)}}$



$\dfrac{2^2 + 7(2) + 10}{2+2} = \dfrac{4 + 14 + 10}{4} = \dfrac{28}{4} = \boxed{b = 7}$

Are the following continuous at x = 2? If not, give the official reason according to the formal definition of continuity.

11. $f(x) = \begin{cases} x^2 & if\ x \leq 2 \\ 3x-2 & if\ x > 2 \end{cases}$ 4, 4

Yes

12. $f(x) = \begin{cases} x^2 & if\ x < 2 \\ 3x-2 & if\ x > 2 \end{cases}$

No, $f(2)$ DNE

13. $f(x) = \begin{cases} x^2 & if\ x \leq 2 \\ 3x+2 & if\ x > 2 \end{cases}$ 4, 8

$\lim\limits_{x \to 2}$ DNE

14. $f(x) = \begin{cases} x^2 & if\ x < 2 \\ 5 & if\ x = 2 \\ 3x-2 & if\ x > 2 \end{cases}$ 4, 4

No, $\lim\limits_{x \to 2} \neq f(2)$