

Series

Test	Series	Converges	Diverges	Comment
nth-Term	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	Only for Divergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$ r < 1$	$ r > 1$	$s = \frac{a}{1-r}$
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n \rightarrow \infty} b_n = L$		$s = b_1 - L$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 < a_{n+1} \leq a_n$ $\lim_{n \rightarrow \infty} a_n = 0$		Remainder: $ R_N \leq a_{N+1}$
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	Remainder: $0 < R_N < \int_N^{\infty} f(x) dx$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } < 1$	$\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } > 1$	Inconclusive if limit=1
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right < 1$	$\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right > 1$	Inconclusive if limit=1
Direct Comparison ($b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ $\sum_{n=1}^{\infty} b_n$ converges	$0 < a_n \leq b_n$ $\sum_{n=1}^{\infty} b_n$ diverges	
Limit Comparison ($b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left \frac{a_n}{b_n} \right = L > 0$ $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n \rightarrow \infty} \left \frac{a_n}{b_n} \right = L > 0$ $\sum_{n=1}^{\infty} b_n$ diverges	

Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
Divergence Test	If $\lim_{k \rightarrow +\infty} u_k \neq 0$, then $\sum u_k$ diverges.	If $\lim_{k \rightarrow +\infty} u_k = 0$, then $\sum u_k$ may or may not converge.
Integral Test	<p>Let $\sum u_k$ be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the general term of the series. If f is decreasing and continuous for $x \geq 1$, then</p> $\sum_{k=1}^{\infty} u_k \quad \text{and} \quad \int_1^{+\infty} f(x) dx$ <p>both converge or both diverge.</p>	<p>This test only applies to series that have positive terms.</p> <p>Try this test when $f(x)$ is easy to integrate.</p>
Comparison Test	<p>Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be series with nonnegative terms such that</p> $a_1 \leq b_1, a_2 \leq b_2, \dots, a_k \leq b_k, \dots$ <p>If $\sum b_k$ converges, then $\sum a_k$ converges, and if $\sum a_k$ diverges, then $\sum b_k$ diverges.</p>	<p>This test only applies to series with nonnegative terms.</p> <p>Try this test as a last resort: other tests are often easier to apply.</p>
Ratio Test	<p>Let $\sum u_k$ be a series with positive terms and suppose that</p> $\rho = \lim_{k \rightarrow +\infty} \frac{u_{k+1}}{u_k}$ <p>(a) Series converges if $\rho < 1$. (b) Series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.</p>	<p>Try this test when u_k involves factorials or kth powers.</p>
Root Test	<p>Let $\sum u_k$ be a series with positive terms such that</p> $\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$ <p>(a) The series converges if $\rho < 1$. (b) The series diverges if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.</p>	<p>Try this test when u_k involves kth powers.</p>
Limit Comparison Test	<p>Let $\sum a_k$ and $\sum b_k$ be series with positive terms such that</p> $\rho = \lim_{k \rightarrow +\infty} \frac{a_k}{b_k}$ <p>If $0 < \rho < +\infty$, then both series converge or both diverge.</p>	<p>This is easier to apply than the comparison test, but still requires some skill in choosing the series $\sum b_k$ for comparison.</p>
Alternating Series Test	<p>If $a_k > 0$ for $k = 1, 2, 3, \dots$, then the series</p> $a_1 - a_2 + a_3 - a_4 + \dots$ $-a_1 + a_2 - a_3 + a_4 - \dots$ <p>converge if the following conditions hold:</p> <p>(a) $a_1 > a_2 > a_3 > \dots$ (b) $\lim_{k \rightarrow +\infty} a_k = 0$</p>	<p>This test applies only to alternating series.</p>
Ratio Test for Absolute Convergence	<p>Let $\sum u_k$ be a series with nonzero terms such that</p> $\rho = \lim_{k \rightarrow +\infty} \frac{ u_{k+1} }{ u_k }$ <p>(a) The series converges absolutely if $\rho < 1$. (b) The series diverges absolutely if $\rho > 1$ or $\rho = +\infty$. (c) The test is inconclusive if $\rho = 1$.</p>	<p>The series need not have positive terms and need not be alternating to use this test.</p>