Day 4 HW:

① $\sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$ $\lim_{n\to\infty} \left| \frac{(-x)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-x)^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(-x)^n}(-x)}{(n+1)\cdot \cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{(-x)^n}} \right| \to \lim_{n\to\infty} \left| \frac{1}{n+1} \cdot -x \right|$

$0 \cdot |-x| < 1$

a) ROC: ∞

b) IOC: $(-\infty, \infty)$

② $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n (x-1)^n$ $\lim_{n\to\infty} \left| \frac{\left(\frac{2}{3}\right)^{n+1}(x-1)^{n+1}}{\left(\frac{2}{3}\right)^n (x-1)^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{\left(\frac{2}{3}\right)^n}\left(\frac{2}{3}\right)\cancel{(x-1)^n}(x-1)}{\cancel{\left(\frac{2}{3}\right)^n}\cancel{(x-1)^n}} \right| = \lim_{n\to\infty} \left| \frac{2}{3}(x-1) \right|$

$-1 < \frac{2}{3}x - \frac{2}{3} < 1$

$-\frac{1}{3} < \frac{2}{3}x < \frac{5}{3}$

$-\frac{1}{2} < x < \frac{5}{2}$

Endpts:

$x = -\frac{1}{2}$: $\left(\frac{2}{3}\right)^n \left(-\frac{3}{2}\right)^n = \left(\frac{2}{3} \cdot -\frac{3}{2}\right)^n$

$= (-1)^n$ div

$x = \frac{5}{2}$: $\left(\frac{2}{3}\right)^n \left(\frac{3}{2}\right)^n = 1^n$ div

a) ROC: $\frac{3}{2}$

b) IOC: $-\frac{1}{2} < x < \frac{5}{2}$

③ $\sum_{n=0}^{\infty} \frac{n!}{2^n} \cdot x^{2n}$ $\lim_{n\to\infty} \left| \frac{(n+1)!\, x^{2(n+1)}}{2^{n+1}} \cdot \frac{2^n}{n! \cdot x^{2n}} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)\cancel{n!}\, \cancel{x^{2n}} \cdot x^2}{\cancel{2^n} \cdot 2} \cdot \frac{\cancel{2^n}}{\cancel{n!}\, \cancel{x^{2n}}} \right| \to \lim_{n\to\infty} \left| (n+1) \cdot \frac{x^2}{2} \right|$

converges at $x=0$

a) ROC: 0

b) IOC: $x=0$

④ $\sum_{n=1}^{\infty} (n+1)! \cdot x^n$ $\lim_{n\to\infty} \left| \frac{(n+2)!\, x^{n+1}}{(n+1)!\, x^n} \right|$

$\lim_{n\to\infty} \left| \frac{(n+2)\cancel{(n+1)!}\, \cancel{x^n} \cdot x}{\cancel{(n+1)!}\, \cancel{x^n}} \right| = \lim_{n\to\infty} \left| (n+2) \cdot x \right|$

converges at $x=0$

a) ROC: 0

b) IOC: $x=0$

⑤ $\frac{1}{1-6x}$ $\frac{1}{1-x} = 1 + x + x^2 + \ldots\, x^n$

$\frac{1}{1-(6x)} = 1 + 6x + (6x)^2 + \ldots\, (6x)^n$

⑥ $\sin(\pi x)$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\, \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$

$\sin(\pi x) = \pi x - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} + \ldots\, \frac{(-1)^n (\pi x)^{2n+1}}{(2n+1)!}$

(7) $1+\cos(2x)$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \quad \frac{(-1)^n \cdot x^{2n}}{(2n)!}$$

$$1+\cos(2x) = 2 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \dots \frac{(-1)^n(2x)^{2n}}{(2n)!} + 1$$

$$2 - \frac{4x^2}{2!} + \frac{16x^4}{4!}$$

(8) $$\frac{1-\sin(x^2)}{x} = \frac{1}{x} - \frac{\sin(x^2)}{x}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \qquad \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(x^2) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} \qquad \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!}$$

$$\frac{(-1)^n \cdot x^{4n+2}}{(2n+1)!}$$

$$1 - \sin(x^2) = 1 - \left(x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!}\right)$$

$$1 - x^2 + \frac{x^6}{3!} - \frac{x^{10}}{5!}$$

$$\frac{1-\sin(x^2)}{x} = \frac{1}{x} - x + \frac{x^5}{3!} - \frac{x^9}{5!}$$

$$\hookrightarrow \frac{1}{x} - \frac{(-1)^n \cdot x^{4n+1}}{(2n+1)!}$$

(9) $\frac{e^x - \cos x}{x}$ or $\frac{e^x}{x} - \frac{\cos x}{x}$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

omit
nth term

$(-)$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

$x + \frac{2x^2}{2!} + \frac{x^3}{3!}$

$\frac{e^x - \cos x}{x} = 1 + \frac{2x}{2!} + \frac{x^2}{3!}$

(10) $-x + \sin x$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ $\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$-x + \sin x = \frac{-x^3}{3!} + \frac{x^5}{5!}$ $\frac{(-1)^{n+1} x^{2n+3}}{(2n+3)!}$

(11) $\cos\sqrt{5x}$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $\frac{(-1)^n \cdot x^{2n}}{(2n)!}$

$\cos\sqrt{5x} = 1 - \frac{(\sqrt{5x})^2}{2!} + \frac{(\sqrt{5x})^4}{4!}$ $\frac{(-1)^n(\sqrt{5x})^{2n}}{(2n)!}$

$1 - \frac{5x}{2!} + \frac{(5x)^2}{4!}$ $\frac{(-1)^n((\sqrt{5x})^2)^n}{(2n)!}$

$\frac{(-1)^n (5x)^n}{(2n)!}$

12 $x \cdot e^{-x^2}$

$e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\frac{x^n}{n!}$

$e^{-x^2} = 1 + -x^2 + \frac{(-x^2)^2}{2!}$ $\frac{(-x^2)^n}{n!}$

$x \cdot e^{-x^2} = x - x^3 + \frac{x^5}{2!}$ $\frac{x \cdot (-x^2)^n}{n!} = \frac{(-1)^n \cdot x^{2n+1}}{n!}$

13 $\sum_{n=1}^{\infty} \frac{x^n}{n}$ $\lim_{n\to\infty} \left| \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x}{(n+1)} \cdot \frac{n}{\cancel{x^n}} \right|$

$\lim_{n\to\infty} \left| \frac{n}{n+1} \cdot x \right|$ $-1 < x < 1$

Endpts:

$x = -1$: $\frac{(-1)^n}{n}$ conv AST

$x = 1$: $\frac{1^n}{n}$ div p-series

a) $[-1, 1)$

b) $f'(x) = \sum \frac{\cancel{n} \cdot x^{n-1}}{\cancel{n}} = \sum x^{n-1}$

$(-1, 1)$

Endpts:

$x = -1$: $(-1)^{n-1} = -1(-1)^n$ div

$x = 1$: $1^{n-1} = -1 \cdot 1^n$ div

c) $\int f(x)dx = \sum \frac{x^{n+1}}{n(n+1)}$

$[-1, 1]$

Endpts:

$x = -1$: $\frac{(-1)^{n+1}}{n(n+1)} = \frac{(-1)^{n+1}}{n(n+1)}$

conv AST

$x = 1$: $\frac{(1)^{n+1}}{n^2+n} = \frac{1}{n^2+n}$ $\frac{1}{n^2}$

$\downarrow$ big

conv DCT conv

14 $\sum_{n=1}^{\infty} \frac{(-1)^n(x-2)^n}{n}$ $\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-2)^{n+1}}{n+1} \cdot \frac{n}{(-1)^n(x-2)^n} \right|$

a)

$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^n}(-1)\cancel{(x-2)^n}(x-2)}{(n+1)} \cdot \frac{n}{\cancel{(-1)^n}\cancel{(x-2)^n}} \right|$

$\lim_{n\to\infty} \left| \frac{n}{n+1} \cdot -1(x-2) \right|$

$-1 < -x+2 < 1$

$-3 < -x < -1$

$3 > x > 1$

Endpts:

$x=1$: $\frac{(-1)^n(-1)^n}{n} = \frac{1}{n}$ div

$x=3$: $\frac{(-1)^n(1)^n}{n}$ conv

$(1, 3]$

b) $f'(x) = \sum \frac{(-1)^n \cdot \cancel{n}(x-2)^{n-1}}{\cancel{n}} = \sum (-1)^n(x-2)^{n-1}$

Endpts:

$x=1$: $(-1)^n(-1)^{n-1} = (-1)^n(-1)^n(-1)^{-1}$ div

$x=3$: $(-1)^n(1)^{n-1} = (-1)^n \cdot 1^n \cdot (1)^{-1}$ div

$(1, 3)$

c) $\int f(x)dx = \sum \frac{(-1)^n(x-2)^{n+1}}{n(n+1)}$

Endpts:

$x=1$: $\frac{(-1)^n(-1)^{n+1}}{n(n+1)} = \frac{-1}{n(n+1)}$ conv by integral

$x=3$: $\frac{(-1)^n(1)^{n+1}}{n(n+1)}$ conv AST

$[1, 3]$