BC Unit 2 FRQ Practice

1. Calc Active:

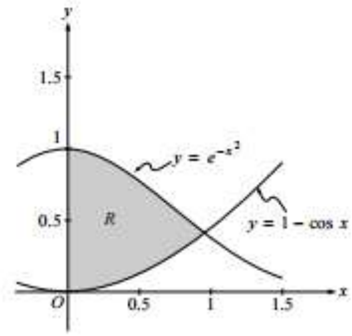
AP Calculus AB–1 / BC–1 2000

Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

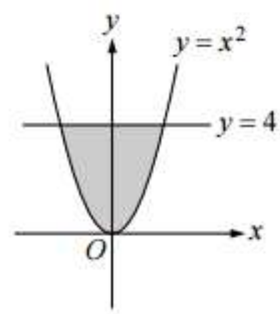
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



2. Calc Active:

AB–2 / BC–2 1999

2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.
   (a) Find the area of R.
   (b) Find the volume of the solid generated by revolving R about the x–axis.
   (c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.



3. Calc Active

AP® CALCULUS AB
2002 SCORING GUIDELINES (Form B)

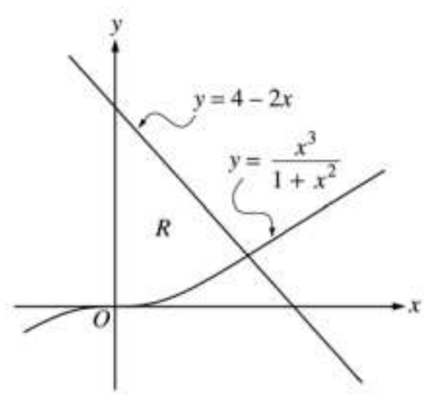
Question 1

Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



4. Calc Active

AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 3

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicates units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

5. Calc Inactive

AP® CALCULUS AB
2003 SCORING GUIDELINES (Form B)

Question 3

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood vessel at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and $B(x)$ is a twice-differentiable function that represents the diameter at that point.

(a) Write an integral expression in terms of $B(x)$ that represents the average radius, in mm, of the blood vessel between $x = 0$ and $x = 360$.

(b) Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your answer.

(c) Using correct units, explain the meaning of $\pi\int_{125}^{275}\left(\frac{B(x)}{2}\right)^2 dx$ in terms of the blood vessel.

(d) Explain why there must be at least one value x, for $0 < x < 360$, such that $B''(x) = 0$.

KEY:

1)

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) $\text{Area} = \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$

$= 0.590$ or 0.591

2 $\begin{cases} 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$

(b) $\text{Volume} = \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

3 $\begin{cases} 2: \text{ integrand and constant} \\ \quad <-1> \text{ each error} \\ 1: \text{ answer} \end{cases}$

(c) $\text{Volume} = \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 $\begin{cases} 2: \text{ integrand} \\ \quad <-1> \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \qquad k\int_c^d (f(x) - g(x))^2\, dx \\ 1: \text{ answer} \end{cases}$

2)

(a) Area $= \int_{-2}^{2} (4 - x^2)\, dx$

$= 2 \int_0^2 (4 - x^2)\, dx$

$= 2 \left[4x - \frac{x^3}{3} \right]_0^2$

$= \frac{32}{3} = 10.666$ or 10.667

$2 \begin{cases} \text{1: integral} \\ \text{1: answer} \end{cases}$

(b) Volume $= \pi \int_{-2}^{2} \left(4^2 - (x^2)^2 \right) dx$

$= 2\pi \int_0^2 (16 - x^4)\, dx$

$= 2\pi \left[16x - \frac{x^5}{5} \right]_0^2$

$= \frac{256\pi}{5} = 160.849$ or 160.850

$3 \begin{cases} \text{1: limits and constant} \\ \text{1: integrand} \\ \text{1: answer} \end{cases}$

(c) $\pi \int_{-2}^{2} \left[(k - x^2)^2 - (k - 4)^2 \right] dx = \frac{256\pi}{5}$

$4 \begin{cases} \text{1: limits and constant} \\ \text{2: integrand} \\ \quad < -1 > \text{ each error} \\ \text{1: equation} \end{cases}$

3.

Region R $\frac{x^3}{1+x^2} = 4 - 2x$ at $x = 1.487664 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right) dx$ $= 3.214$ or 3.215	2 $\begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^A \left((4-2x)^2 - \left(\frac{x^3}{1+x^2}\right)^2\right) dx$ $= 31.884$ or 31.885 or 10.149π	3 $\begin{cases} 2: \text{integrand and constant} \\ \quad <-1> \text{ each error} \\ 1: \text{answer} \end{cases}$
(c) Volume $= \int_0^A \left(4 - 2x - \frac{x^3}{1+x^2}\right)^2 dx$ $= 8.997$	3 $\begin{cases} 2: \text{integrand} \\ \quad <-1> \text{ each error} \\ \quad \text{note: } 0/2 \text{ if not of the form} \\ \qquad k\int_c^d (f(x) - g(x))^2\, dx \\ 1: \text{answer} \end{cases}$

4)

(a) Midpoint Riemann sum is

$$10 \cdot [v(5) + v(15) + v(25) + v(35)]$$
$$= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$$

The integral gives the total distance in miles that the plane flies during the 40 minutes.

3 : { 1 : $v(5) + v(15) + v(25) + v(35)$; 1 : answer; 1 : meaning with units }

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.

2 : { 1 : two instances; 1 : justification }

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2

1 : answer with units

(d) Average velocity $= \frac{1}{40}\int_0^{40} f(t)\, dt$
$= 5.916$ miles per minute

3 : { 1 : limits; 1 : integrand; 1 : answer }

5)

(a) $\frac{1}{360}\int_0^{360} \frac{B(x)}{2}\,dx$	$2: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$
(b) $\frac{1}{360}\left[120\left(\frac{B(60)}{2} + \frac{B(180)}{2} + \frac{B(300)}{2}\right)\right] =$ $\frac{1}{360}[60(30 + 30 + 24)] = 14$	$2: \begin{cases} 1: B(60) + B(180) + B(300) \\ 1: \text{answer} \end{cases}$
(c) $\frac{B(x)}{2}$ is the radius, so $\pi\left(\frac{B(x)}{2}\right)^2$ is the area of the cross section at x. The expression is the volume in mm^3 of the blood vessel between 125 mm and 275 mm from the end of the vessel.	$2: \begin{cases} 1: \text{volume in mm}^3 \\ 1: \text{between } x = 125 \text{ and } \\ \quad x = 275 \end{cases}$
(d) By the MVT, $B'(c_1) = 0$ for some c_1 in $(60, 180)$ and $B'(c_2) = 0$ for some c_2 in $(240, 360)$. The MVT applied to $B'(x)$ shows that $B''(x) = 0$ for some x in the interval (c_1, c_2).	$3: \begin{cases} 2: \text{explains why there are two} \\ \quad \text{values of } x \text{ where } B'(x) \text{ has} \\ \quad \text{the same value} \\ 1: \text{explains why that means} \\ \quad B''(x) = 0 \text{ for } 0 < x < 360 \end{cases}$ Note: max 1/3 if only explains why $B'(x) = 0$ at some x in $(0, 360)$.