Applications of Derivatives Review

Name: Key

Problems from Textbook	
Page #	Problems
160	105, 107
242	5, 11, 15, 17, 19, 23, 24,25

SAMPLE AP QUESTIONS

1) The slope of the curve $y^3 - xy^2 = 4$ at the point where y = 2 is
   a) -2
   b) ¼
   c) −½
   d) ½
   e) 2

2) The slope of the curve $y^2 - xy - 3x = 1$ at the point (0, -1) is
   a) -1
   b) -2
   c) 1
   d) 2
   e) -3

3) The equation of the tangent to the curve $y = xsinx$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is
   a) $y = x - \pi$
   b) $y = \pi/2$
   c) $y = \pi - x$
   d) $y = x + \pi/2$
   e) $y = x$

4) The tangent to the curve of $y = xe^{-x}$ is horizontal when x is equal to
   a) 0
   b) 1
   c) -1
   d) 1/e
   e) None of these

5) The minimum value of the slope of the curve $y = x^5 + x^3 - 2x$ is
   a) 0
   b) 2
   c) 6
   d) -2
   e) None of these

6) The equation of the tangent to the hyperbola $x^2 - y^2 = 12$ at the point (4, 2) on the curve
   a) $x - 2y + 6 = 0$
   b) $y = 2x$
   c) $y = 2x - 6$
   d) $y = \frac{x}{2}$
   e) $x + 2y = 6$

7) The function $f(x) = x^4 - 4x^2$ has
   a) One relative minimum and two relative maximum
   b) One relative minimum and one relative maximum
   c) Two relative maxima and no relative minimum
   d) Tow relative minima and no relative maximum
   e) Two relative minima and one relative maximum

8) The number of inflection points of the curve in Question 7 is
   a) 0
   b) 1
   c) 2
   d) 3
   e) 4

9) The maximum value of the function $y = -4\sqrt{2 - x}$ is
   a) 0
   b) -4
   c) 2
   d) -2
   e) None of these

10) The total number of maximum and minimum points of the function whose derivative, for all x, is given by $f'(x) = x(x-3)^2(x+1)^4$ is
   a) 0
   b) 1
   c) 2
   d) 3
   e) None of these

11) A circular conical reservoir, vertex down, has depth 20 ft and radius of the top 10 ft. Water is leaking out so that the surface is falling at the rate of ½ ft/hr. The rate, in cubic feet per hour, at which the water is leaving the reservoir when the water is 8 ft deep is

a) 4π c) 16π e) $1/(8\pi)$
b) 8π d) $1/(4\pi)$

12) A local minimum value of the function $y = \frac{e^x}{x}$ is

a) 1/e c) -1 e) 0
b) 1 d) e

13) The point of the curve $y = \sqrt{2x+1}$ at which the normal is parallel to the line $y = -3x + 6$ is

a) (4, 3) c) $(1, \sqrt{3})$ e) $(2, \sqrt{5})$
b) (0, 1) d) (4, -3)

14) The number of vertical tangents to the graph of $y^2 = x - x^3$ is

a) 4 c) 2 e) 0
b) 3 d) 1

15) $\lim_{h\to 0} \frac{(2+5)^5 - 2^5}{h}$

a) 0 c) 32 e) 160
b) 1 d) 80

16) How many critical points does the function $f(x) = |x^3 - 2x|$ have over its entire domain?

a) 2 c) 4 e) Infinitely many
b) 3 d) 5

17) The function g is continuous on the interval [-1, 2] and differentiable (-1, 2). If g(-1) =2 and g(2)=-4, which of the following statements in NOT necessarily true?

a) There exist a value c on (-1, 2) such that f(c) = 0
b) There exist a value c on (-1, 2) such that f'(c) = 0
c) There exist a value c on (-1, 2) such that f(c) =-3
d) There exist a value c on (-1, 2) such that f'(c) =-2
e) There exist a value c on [1, 2] such that f(c) ≥f(x) for all x on [-1, 2]

x	f	g	f'	g'
1	3	4	2/3	-5/2
2	4	2	4/3	-3/2
4	8	1	8/3	1/2

18) If f(x) and g(x) are differentiable function with values as given in the chart above, and k(x) =f(g(x^2)), what is k'(2)?

a) 1/3 c) 4/3 e) None of these
b) 2/3 d) 16/3

19) For what value of c on [0, 1] is the tangent to graph of $f(x) = e^x - x^2$ parallel to the secant line? (Calculator)

a) -0.248 c) 0.500 e) 0.718
b) 0.351 d) 0.693

20) A 26-foot ladder leans against a building so that its foot moves away from the building at the rate of 3 ft/sec. When the foot of the ladder is 10 feet from the building, the top is moving down at the rate of r feet per second, where r is

a) 46/3 d) 5/2
b) ¾ e) 4/5
c) 5/4

Apps of Der Review:

1) $y^3 - xy^2 = 4$

D $3y^2\frac{dy}{dx} - (x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1) = 0$

$\frac{dy}{dx}(3y^2 - 2xy) = y^2$

$\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}$

$\left.\frac{dy}{dx}\right|_{y=2} = \frac{4}{12 - 2(1)(2)}$

$= \frac{4}{8} = \boxed{\frac{1}{2}}$

$2^3 - x(2)^2 = 4$

$8 - 4x = 4$

$-4x = -4$

$x = 1$

2) $y^2 - xy - 3x = 1 \quad (0, -1)$

A $2y\frac{dy}{dx} - (x\frac{dy}{dx} + y(1)) - 3 = 0$

$\frac{dy}{dx}(2y - x) = y + 3$

$\frac{dy}{dx} = \frac{y+3}{2y-x}$

$\left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{-1+3}{2(-1)-0} = \frac{2}{-2} = \boxed{-1}$

3) $y = x \sin x \quad \left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

E $y' = x(\cos x) + \sin x \cdot 1$

$y'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$

$y'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}(0) + 1$

$y'\left(\frac{\pi}{2}\right) = 1$

$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

$y - \frac{\pi}{2} = x - \frac{\pi}{2}$

$\boxed{y = x}$

4) $y' = x \cdot (-e^{-x}) + e^{-x}(1)$

B $y' = -xe^{-x} + e^{-x} = 0$

$e^{-x}(-x+1) = 0$

$x = 1$

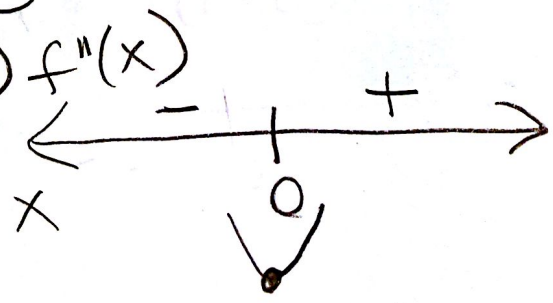
5) $y = x^5 + x^3 - 2x$

A $y' = 5x^4 + 3x^2 - 2 = 0$

$y'' = 20x^3 + 6x = 0$

$2x(10x^2 + 3) = 0$

$x = 0$



$f(0) = 0$

6) $x^2 - y^2 = 12 \quad (4,2)$

C $2x - 2y\frac{dy}{dx} = 0$

$-2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{x}{y} = \frac{4}{2} = 2$

$y - 2 = 2(x-4)$

$y - 2 = 2x - 8$

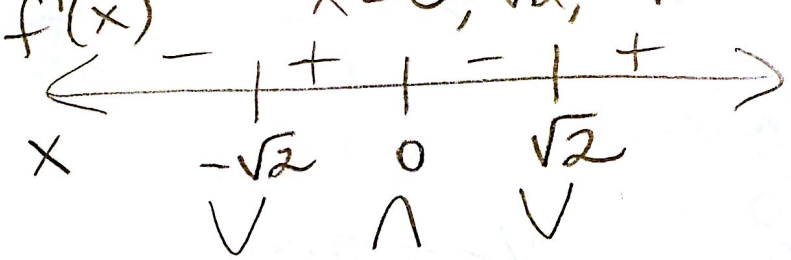
$y = 2x - 6$

7) $f(x) = x^4 - 4x^2$

E $f'(x) = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$x = 0, \sqrt{2}, -\sqrt{2}$

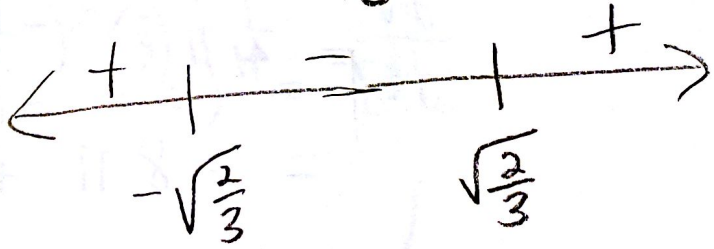


8) $f''(x) = 12x^2 - 8 = 0$

C $x^2 = \frac{8}{12}$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$



9) $y = -4(2-x)^{1/2}$

A $y' = -2(2-x)^{-1/2} \cdot -1 = 2 \cdot \frac{1}{2-x} = \frac{2}{2-x}$

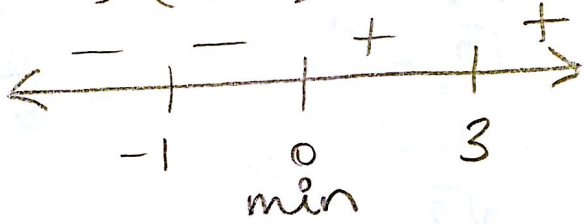
$x = 2$

$x = 2$

$y = 0$

10) $f'(x) = x(x-3)^2(x+1)^4$

B



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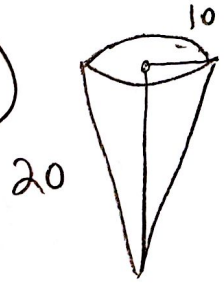


B

$\frac{r}{h} = \frac{10}{20}$

$20r = 10h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$

$V = \frac{1}{12}\pi h^3$

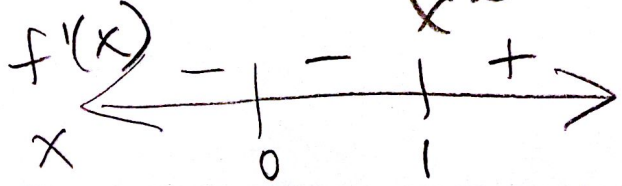
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi(8)^2\left(-\frac{1}{2}\right)$

$= -8\pi \text{ ft}^3/\text{hr}$

12) $y = \frac{e^x}{x}$

D $y' = \frac{x \cdot e^x - e^x \cdot 1}{x^2} = \frac{e^x(x-1)}{x^2}$ $x = 1, 0$



$x = 1$ $\frac{e^1}{1} = e$

13) $y = (2x+1)^{1/2}$ tan line

A $y' = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+1}} = \frac{1}{3}$

$y = \sqrt{9} = 3$

$(4, 3)$

$3 = \sqrt{2x+1}$

$9 = 2x+1$

$x = 4$

14) $y^2 = x - x^3$

B $2y\frac{dy}{dx} = 1 - 3x^2$

$\frac{dy}{dx} = \frac{1-3x^2}{2y}$

$y = 0$

$0 = x - x^3$

$0 = x(1-x^2)$

$x = 0, 1, -1$

$(0, 0)$

$(1, 0)$

$(-1, 0)$

15) $\lim_{h \to 0} \frac{(2+5)^5 - 2^5}{h}$

D

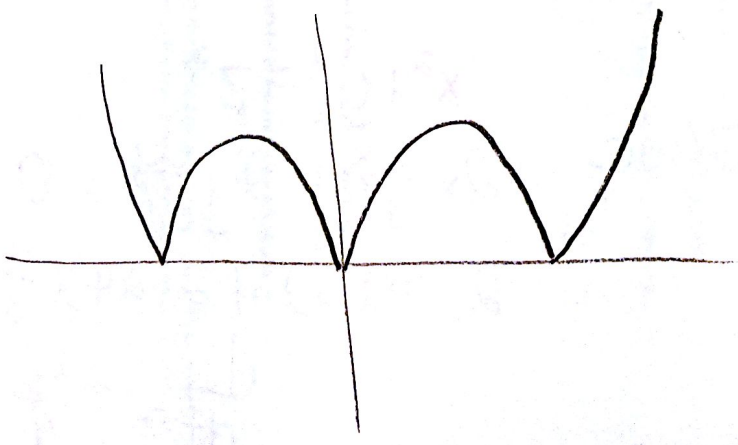
$f(x) = x^5$

$f'(x) = 5x^4$

$f'(2) = 5(2)^4 = 80$

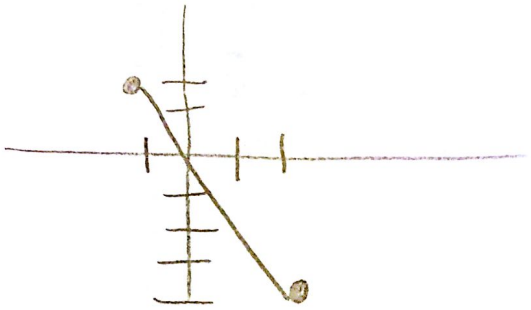
16)

D



5 critical values

17) B $\frac{-4-2}{2-(-1)} = \frac{-6}{3} = -2$



slope $\neq 0$

18) C $f(g(x^2)) = K(x)$

$K'(x) = f'(g(x^2)) \cdot g'(x^2) \cdot 2x$

$f'(g(4)) \cdot g'(4) \cdot 4$

$f'(1) \cdot \frac{1}{2} \cdot 4$

$\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{4}{1} = \frac{4}{3}$

19) B $y = e^x - x^2$

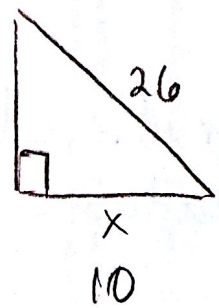
$y' = e^x - 2x$

$\frac{f(1) - f(0)}{1-0} = \frac{1.718\ldots - 1}{1} = .718$

$e^x - 2x = .718$

$x = .351$

20)



$\frac{dx}{dt} = 3$ ft/sec

$x = 10$

$\frac{dy}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(10)(3) + 2(24)\left(\frac{dy}{dt}\right) = 0$

$\frac{dy}{dt} = -\frac{5}{4}$ ft/s

105) $x\sin y = y\cos x$

$$x\cos y\frac{dy}{dx} + \sin y = y(-\sin x) + \cos x\frac{dy}{dx}$$

$$(x\cos y - \cos x)\frac{dy}{dx} = -y\sin x - \sin y$$

$$\frac{dy}{dx} = \frac{-y\sin x - \sin y}{x\cos y - \cos x}$$

107) $x^2 + y^2 = 20 \quad (2,4)$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

T: $\frac{-2}{4} = \frac{-1}{2}$

$y - 4 = -\frac{1}{2}(x-2)$

$2(y-4 = -\frac{1}{2}x + 1)$

$2y - 8 = -x + 2$

$x + 2y - 10 = 0$

N: 2

$y - 4 = 2(x-2)$

$y - 4 = 2x - 4$

$y - 2x = 0$

5) $f(x) = (x-2)(x^2+6x+9)$

$f(x) = x^3 + 6x^2 + 9x - 2x^2 - 12x - 18$

$f(x) = x^3 + 4x^2 - 3x - 18$

$f'(x) = 3x^2 + 8x - 3 = 0$

$(3x - 1)(x + 3) = 0$

$x = -3, \frac{1}{3}$

11) $f(x) = x - \cos x \quad \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\frac{f(\frac{\pi}{2}) - f(\frac{-\pi}{2})}{\frac{\pi}{2} - (\frac{-\pi}{2})} = \frac{\frac{\pi}{2} + \frac{\pi}{2}}{\frac{\pi}{2} + \frac{\pi}{2}} = 1$$

$f'(x) = 1 + \sin x = 1$

$\sin x = 0$

$x = 0$

15) $f(x) = (x-1)^2(x-3)$

$f(x) = (x^2-2x+1)(x-3)$

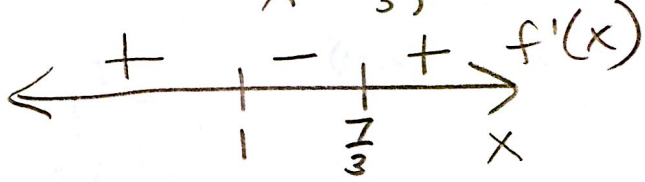
$f(x) = x^3 - 3x^2 - 2x^2 + 6x + x - 3$

$f(x) = x^3 - 5x^2 + 7x - 3$

$f'(x) = 3x^2 - 10x + 7 = 0$

$(3x-7)(x-1) = 0$

$x = \frac{7}{3}, 1$



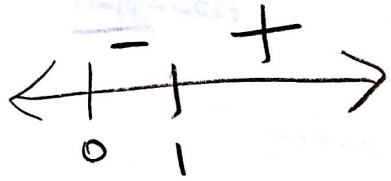
17) $h(x) = \sqrt{x}(x-3)$, $x > 0$

$h(x) = x^{1/2}(x-3) = x^{3/2} - 3x^{1/2}$

$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$

$\frac{\sqrt{x}}{\sqrt{x}} \cdot \frac{3\sqrt{x}}{2} - \frac{3}{2\sqrt{x}} = 0$

$\frac{3x}{2\sqrt{x}} - \frac{3}{2\sqrt{x}} = 0$



$3x - 3 = 0$

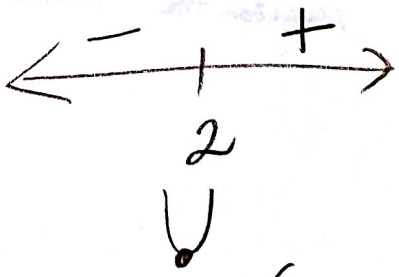
$x = 1$

19) $h(t) = \frac{1}{4}t^4 - 8t$

$h'(t) = t^3 - 8 = 0$

$t^3 = 8$

$t = 2$



rel min: $(2, -12)$

23) $f(x) = x + \cos x \quad [0, 2\pi]$

$f'(x) = 1 - \sin x$

$f''(x) = -\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

POI: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

CC↑ $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ CC↓ $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

24) $f(x) = (x+2)^2(x-4)$

$f(x) = (x^2 + 4x + 4)(x-4)$

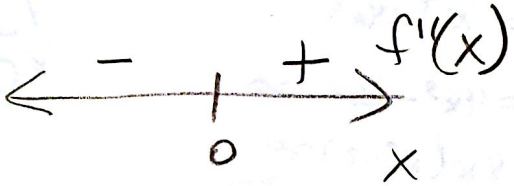
$f(x) = x^3 - 4x^2 + 4x^2 - 16x + 4x - 16$

$f(x) = x^3 - 12x - 16$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x = 0$

$x = 0$



POI: $(0, -16)$

CC↓ : $(-\infty, 0)$

CC↑ : $(0, \infty)$

25) $g(x) = 2x^2(1 - x^2)$

$g(x) = 2x^2 - 2x^4$

$g'(x) = 4x - 8x^3$

$g''(0) = 4$ CC↑

$g''\left(\frac{\sqrt{2}}{2}\right) = -8$ CC↓ $g''\left(-\frac{\sqrt{2}}{2}\right) = -8$ CC↓

$g'(x) = 4x - 8x^3 = 0$

$4x(1 - 2x^2) = 0$

$x = 0 \quad x = \pm\frac{\sqrt{2}}{2}$

$-2x^2 = -1$

$x^2 = \frac{1}{2}$

MAX: $\left(\frac{\sqrt{2}}{2}, \frac{1}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$ MIN: $(0, 0)$