

Key

A point moves on the x -axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$.

At what value of t does v attain its maximum?

- (A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$
(E) There is no maximum value for v .

$$\frac{d}{dx}(\ln e^{2x}) =$$

- (A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

- (A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

$$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$$

- (A) $\ln \sqrt{2}$ (B) $\ln \frac{\pi}{4}$ (C) $\ln \sqrt{3}$ (D) $\ln \frac{\sqrt{3}}{2}$ (E) $\ln e$

What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

- (A) $(-1, 0)$ (B) $(0, 0)$ (C) $(0, 1)$ (D) $(1, \frac{\pi}{4})$ (E) $(1, \frac{\pi}{2})$

If $F(x) = \int_0^x e^{-t^2} dt$, then $F'(x) =$

- (A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\frac{e^{-x^2+1}}{-x^2+1} - e$
(D) $e^{-x^2} - 1$ (E) e^{-x^2}

An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

- (A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$
(D) $y = 0$ (E) $\pi x - 2y = 0$

At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

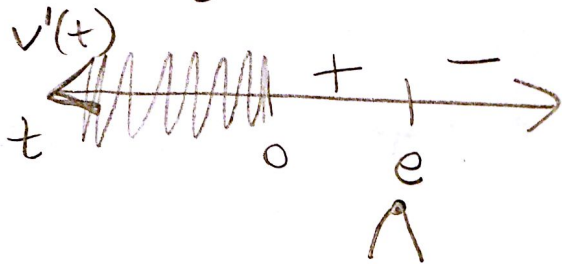
- (A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

Test Review #1:

$$1) \quad v = \frac{\ln t}{t} \quad v' = \frac{t \cdot \frac{1}{t} - \ln t \cdot 1}{t^2}$$

$$v' = \frac{1 - \ln t}{t^2} = 0$$

$$\begin{aligned} 1 - \ln t &= 0 \\ -\ln t &= -1 \\ \ln t &= 1 \\ t &= e \end{aligned}$$



$$2) \quad \frac{d}{dx} (\ln e^{2x}) = \frac{d}{dx} (2x \ln e) = \frac{d}{dx} (2x) = 2$$

$$3) \quad \sin x = e^y \quad \frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$$

$$\cos x = e^y \cdot \frac{dy}{dx}$$

$$\cos x = \sin x \cdot \frac{dy}{dx}$$

$$4) \quad \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x} dx \quad \begin{aligned} u &= \sin x \\ du &= \cos x dx \end{aligned}$$

$$u\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$u\left(\frac{\pi}{2}\right) = 1$$

$$\begin{aligned} \int_{\frac{\sqrt{2}}{2}}^1 \frac{1}{u} du &= \ln|u| \Big|_{\frac{\sqrt{2}}{2}}^1 = \cancel{\ln 1} - \ln \frac{\sqrt{2}}{2} \rightarrow \ln \frac{2}{\sqrt{2}} \\ &= \ln \left(\frac{\sqrt{2}}{2}\right)^{-1} = \ln \sqrt{2} \\ &= \ln \left(\frac{2}{\sqrt{2}}\right) \end{aligned}$$

$$5) y = (x+1) \cdot \arctan x$$

$$y' = (x+1) \cdot \frac{1}{1+x^2} + \arctan x \cdot 1$$

$$y' = \frac{x+1}{x^2+1} + \arctan x$$

$$y'' = \frac{(x^2+1)(1) - (x+1)(2x)}{(x^2+1)^2} + \frac{1}{1+x^2} \left(\frac{x^2+1}{x^2+1} \right)$$

$$y'' = \frac{\cancel{x^2+1} - \cancel{2x^2} - 2x + \cancel{x^2+1}}{(x^2+1)^2}$$

$$y'' = \frac{-2x+2}{(x^2+1)^2} = 0 \quad -2(x-1)=0$$

$$x=1$$

$$y'' \begin{array}{c} \leftarrow + \quad | \quad - \rightarrow \\ x \quad \quad 1 \end{array}$$

$$y(1) = (1+1) \cdot \arctan(1) = 2 \cdot \frac{\pi}{4} = \frac{\pi}{2}$$

$$x \quad \left(1, \frac{\pi}{2}\right)$$

$$6) F(x) = \int_0^x e^{-t^2}$$

$$F'(x) = e^{-x^2}$$

$$7) y = \arcsin\left(\frac{x}{2}\right)$$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{1 - \frac{x^2}{4}}}$$

$$y'(0) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$y(0) = \arcsin\left(\frac{0}{2}\right) = 0$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$

$$2y = x$$

$$x - 2y = 0$$

$$8) f(x) = x^2 + e^{-2x}$$

$$f'(x) = 2x + e^{-2x} \cdot (-2)$$

$$f'(x) = 2x - 2e^{-2x}$$

$$f'(0) = 0 - 2e^0$$

$$f'(0) = 0 - 2 = -2$$

f is dec

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a) $\lim_{x \rightarrow 0^+} \frac{x}{\ln x} = 0 \neq f(0) = 1$

f is not cont. at $x=0$. f is cont. at all other values of x except $x=1, -1$ where it is not defined

b) $f(-x) = \frac{-(-x)}{\ln(-(-x))} = \frac{x}{\ln x} = f(x)$
 $x > 0$

$x < 0$ $f(-x) = \frac{-x}{\ln(-x)} = f(x)$

(i) symmetric about y -axis since $f(-x) = f(x)$ for all x

(ii) not symmetric about origin since $f(-x) \neq -f(x)$

c) and d)

$f'(x) = \frac{\ln x - 1}{(\ln x)^2} \quad x > 0$

$f'(x) = \frac{-\ln(-x) + 1}{(\ln(-x))^2} \quad x < 0$

• For $x > 0$, $f'(x) = 0$ at $x = e$ so

(e, e) is rel min

• By symmetry, $(-e, e)$ is rel min also

$f''(x) = \frac{\frac{1}{2}(\ln x)^2 - (\ln x - 1)\left(\frac{2}{x}\right)\ln x}{(\ln x)^4} \quad x > 0$

• $f(x) < 0$ for $-1 < x < 0$ and $0 < x < 1$
 $(0, 1)$ is rel max

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$$a) f'(x) = e^{-2x} - 2xe^{-2x} = e^{-2x}(1-2x)$$

$$f'(x) > 0 \text{ when } 1-2x > 0$$

$$\text{inc } 0 \leq x < \frac{1}{2}$$

$$f'(x) < 0 \text{ when } 1-2x < 0$$

$$\text{dec } \frac{1}{2} < x \leq 10$$

$$b) f'(x) = 0 \Rightarrow e^{-2x}(1-2x) = 0$$

$$\text{crit pt } x = \frac{1}{2}$$

$$0 < x < \frac{1}{2} \Rightarrow f'(x) > 0$$

$$\frac{1}{2} < x < 10 \Rightarrow f'(x) < 0$$

graph of f inc then dec on
 $0 \leq x \leq 10$ so abs max is $(\frac{1}{2}, \frac{1}{2e})$

Abs min at endpt:

$$f(0) = 0 \quad f(10) = \frac{10}{e^{20}}$$

$$\text{Abs min: } (0, 0)$$

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a) $a = 10e^{2t}$

$$v = \int 10e^{2t} dt = 5e^{2t} + C$$

$$v(0) = 5 \Rightarrow C = 0$$

$$v = 5e^{2t}$$

b) $v = 5$ when $t = 0$

$$v = 15 \Rightarrow 5e^{2t} = 15$$

$$t = \frac{1}{2} \ln 3$$

$$\begin{aligned} \text{Dist} &= \int_0^{\frac{1}{2} \ln 3} 5e^{2t} dt = \left. \frac{5}{2} e^{2t} \right|_0^{\frac{1}{2} \ln 3} \\ &= \frac{5}{2} e^{\ln 3} - \frac{5}{2} = 5 \end{aligned}$$

c) $s = \int 5e^{2t} dt = \frac{5}{2} e^{2t} + C$

$$s(0) = 0 \Rightarrow C = -\frac{5}{2}$$

$$s = \frac{5}{2} e^{2t} - \frac{5}{2}$$