AP Calculus AB

Limits Review

Name: Key

Evaluate the following limits. You may or may not need to use L'Hopital's Rule.

B

69AB

6. What is $\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

$f(x) = 8x^8$
$f'(x) = 64x^7$

$f'(\tfrac{1}{2}) = 64(\tfrac{1}{2})^7 = \frac{1}{2}$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

D

69BC

28. What is $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x}$?

$\lim_{x\to 0} e^{2x}-1 = 0$
$\lim_{x\to 0} \tan x = 0$

L'Hopital:
$\lim_{x\to 0} \frac{2e^{2x}}{\sec^2 x} = \frac{2}{1} = 2$

(A) −1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

C

73AB

23. $\lim_{h\to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

$\lim_{h\to 0} \frac{\ln\left(\frac{2+h}{2}\right)}{h}$

$\lim_{h\to 0} \ln\left(\frac{2+h}{2}\right) = 0$
$\lim_{h\to 0} h = 0$

L'Hopital:
$\lim_{h\to 0} \frac{\frac{2}{2+h}\cdot\frac{1}{2}}{1} = \frac{1}{2}$

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

E

73BC

37. $\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

$1-(\cos(2x))^2$

$\lim_{x\to 0} 1-\cos^2(2x) = 0$
$\lim_{x\to 0} x^2 = 0$

L'Hopital:
$\lim_{x\to 0} \frac{-2(-\sin(2x))\cdot 2}{2x} \to \frac{0}{0}$

$\frac{+4\sin 2x}{2x}$

L'H: $\lim_{x\to 0} \frac{+4\cos 2x \cdot 2}{2} = \frac{+8}{2} = +4$

(A) −2 (B) 0 (C) 1 (D) 2 (E) 4

D

85AB

5. $\lim_{n\to\infty} \frac{4n^2}{n^2+\cancel{10{,}000n}}$ is

$\frac{4n^2}{n^2} = 4$

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

D

85AB

37. $\lim_{x\to 0}(x\csc x)$ is

$\lim_{x\to 0} \frac{x}{\sin x}$

L'H: $\lim_{x\to 0} \frac{1}{\cos x} = 1$

(A) $-\infty$ (B) −1 (C) 0 (D) 1 (E) ∞

C

85BC

23. $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is

$\frac{\sqrt{(1+h)^5+8}}{1}$ $\frac{\sqrt{9}}{1} = 3$

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

D

85BC

29. $\lim_{x\to\frac{\pi}{4}} \frac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is

$\frac{\cos(x-\pi/4)}{1} = 1$

(A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) 1 (E) nonexistent

C

85BC

38. $\lim_{x\to\infty} \left(1+5e^x\right)^{\frac{1}{x}}$ is

$y = \lim_{x\to\infty} (1+5e^x)^{1/x}$

$\ln y = \lim_{x\to\infty} \frac{1}{x}\ln(1+5e^x)$

$\ln y = \lim_{x\to\infty} \frac{\ln(1+5e^x)}{x}$

$\ln y = \lim_{x\to\infty} \frac{\frac{1}{1+5e^x}\cdot 5e^x}{1} = \frac{5e^x}{1+5e^x}$

1

$\ln y = 1$

$y = e$

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent

B

88AB

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

$\frac{\cos x}{1} = 1$

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

B

88AB

29. The $\lim_{h\to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

$f(x) = \tan(3x)$

$f'(x) = \sec^2(3x)\cdot 3$

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

D

88BC

10. $\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$ is

$f(x) = \sin x$

$f'(x) = \cos x$

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

A

88BC

35. If k is a positive integer, then $\lim_{x\to+\infty} \frac{x^k}{e^x}$ is

fast!

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

93AB

3. $\lim_{n\to\infty} \frac{3n^3-5n}{n^3-2n^2+1}$ is

(A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

D

93AB

29. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$ is $\to 0$, $\to 0$, $2(\sin\theta)^2$

L'H: $\lim_{\theta\to 0} \frac{\sin\theta}{4(\sin\theta)'\cdot\cos\theta}$

L'H: $\frac{\cos\theta}{4(\sin\theta\cdot -\sin\theta+\cos\theta\cdot\cos\theta)}$

$\frac{1}{4(0+1)} = \frac{1}{4}$

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

C

93BC

2. If $f(x)=2x^2+1$, then $\lim_{x\to 0} \frac{f(x)-f(0)}{x^2}$ is

$\frac{2x^2 \cancel{+1} \cancel{-1}}{x^2}$ $\frac{2x^2}{x^2} = 2$

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

C

98BC

28. $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2-1}$ is

$\frac{0}{0}$ $\frac{e^{x^2}}{2x}$ $\frac{e}{2}$

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

C

93BC

42. $\lim_{x\to 0}(1+2x)^{\csc x} =$

$\ln y = \csc x \cdot \ln(1+2x)$

$\frac{\ln(1+2x)}{\sin x}$

$\frac{\frac{1}{1+2x}\cdot 2}{\cos x} = \frac{2}{1} = 2$

$\ln y = 2$

$y = e^2$

(A) 0 (B) 1 (C) 2 (D) e (E) e^2

E

97BC

16. $\lim_{h\to 0} \frac{e^h-1}{2h}$ is

$\frac{e^h}{2}$ $\frac{1}{2}$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

B

98AB

83. If $a\neq 0$, then $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$ is

$\frac{\cancel{(x+a)(x-a)}}{\cancel{(x+a)(x-a)}(x^2+a^2)}$ $\frac{1}{a^2+a^2}$ $\frac{1}{2a^2}$

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

B