AP Calculus BC
Chapter 8 – AP Exam Problems

L'Hopital's Rule

1. Find $\lim_{x\to 0} x\csc x =$

   A) $-\infty$ B) -1 C) 0 D) 1 E) ∞

2. If $f'(x)=\cos x$ and $g'(x)=1$ for all x, and if $f(0)=g(0)=0$, then $\lim_{x\to 0}\dfrac{f(x)}{g(x)}=$

   A) $\dfrac{\pi}{2}$ B) 1 C) 0 D) -1 E) nonexistent

3. Find $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{2\sin^2\theta}=$

   A) 0 B) $\dfrac{1}{8}$ C) $\dfrac{1}{4}$ D) 1 E) nonexistent

4. Find $\lim_{h\to 0}\dfrac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h}$

   A) 0 B) 1 C) 3 D) $2\sqrt{2}$ E) nonexistent

5. Find $\lim_{x\to\frac{\pi}{4}}\dfrac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}$ is

   A) 0 B) $\dfrac{1}{\sqrt{2}}$ C) $\dfrac{\pi}{4}$ D) 1 E) nonexistent

6. Let f and g be continuous functions that are differentiable for all real numbers, with $g(x)\neq 0$ for $x\neq 0$. If $\lim_{x\to 0} f(x)=\lim_{x\to 0} g(x)=0$ and $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0}\dfrac{f(x)}{g(x)}$ is

   A) 0 C) $\lim_{x\to 0}\dfrac{f'(x)}{g'(x)}$ E) nonexistent

   B) $\dfrac{f'(x)}{g'(x)}$ D) $\dfrac{f'(x)g(x)-f(x)g'(x)}{\left(f(x)\right)^2}$

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7. Find $\lim_{x\to 1}\dfrac{\int_1^x e^{t^2}}{x^2-1}$

A) 0 B) 1 C) $\dfrac{e}{2}$ D) e E) nonexistent

8. Find $\lim_{x\to 0}(1+2x)^{\csc x}=$

A) 0 B) 1 C) 2 D) e E) e^2

9. Find $\lim_{x\to\infty}\left(1+5e^x\right)^{1/x}=$

A) 0 B) 1 C) e D) e^5 E) nonexistent

10. If k is a positive integer, then $\lim_{x\to+\infty}\dfrac{x^k}{e^x}=$

A) 0 B) 1 C) e D) $k!$ E) nonexistent

11. Find $\lim_{x\to 0}\dfrac{e^x-\cos x-2x}{x^2-2x}$

A) -1/2 B) 0 C) ½ D) 1 E) nonexistent

12. Find $\lim_{x\to 0}\dfrac{\sin x\cos x}{x}$

A) -1 B) 0 C) 1 D) $\dfrac{\pi}{4}$ E) nonexistent

13. Find $\lim_{x\to 3}\dfrac{e^{x^2}-e^9}{x-3}$

A) 0 B) $\dfrac{e^9}{9}$ C) $3e^9$ D) $6e^9$ E) nonexistent

14. Find $\lim_{x\to 1}\dfrac{\int_1^x \sin t\,dt}{x^2-1}$

A) 0 B) 1 C) $\dfrac{\sin 1}{2}$ D) 2 E) nonexistent

15. Find $\lim_{x\to 0}\dfrac{\cos x - e^x}{\ln(1+x)}$

A) -1 B) 0 C) 1 D) e E) nonexistent

16. Let $f(x)$ be a continuous function with the properties that $\lim_{x\to\infty} f(x)=\infty$ and $\lim_{x\to\infty} f'(x)=3$.

What is the value of $\lim_{x\to\infty}\left[f(x)\right]^{\frac{1}{x}}$?

A) 0 B) 1 C) 3 D) e^3 E) nonexistent

17. Let $f(x)$ be a continuous function with the properties that $\lim_{x\to 0} f(x)=\infty$ and $\lim_{x\to 0} f'(x)=4$.

What is the value of $\lim_{x\to 0}\left[e^x\right]^{f(x)}$?

A) 0 B) 1 C) 4 D) e^4 E) nonexistent

Improper Integrals

18. Evaluate $\int_2^{+\infty}\dfrac{dx}{x^2}$

A) $\dfrac{1}{2}$ B) $\ln 2$ C) 1 D) 2 E) nonexistent

19. Evaluate $\int_4^{\infty}\dfrac{-2x}{\sqrt[3]{9-x^2}}\,dx$

A) $7^{2/3}$ C) $9^{2/3}+7^{2/3}$ E) nonexistent

B) $\dfrac{3}{2}\left(7^{2/3}\right)$ D) $\dfrac{3}{2}\left(9^{2/3}+7^{2/3}\right)$

20. Evaluate $\int_0^{\infty} x^2 e^{-x^3}\,dx$

A) $-\frac{1}{3}$ B) 0 C) $\frac{1}{3}$ D) 1 E) divergent

21. What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}dx$ converges?

A) $p < -1$ B) $p > 0$ C) $p > ½$ D) $p > 1$ E) There are no values.

22. Let R be the region between the graph of $y = e^{-2x}$ and the x – axis for $x \geq 3$. The area of R is

A) $\frac{1}{2e^6}$ B) $\frac{1}{e^6}$ C) $\frac{2}{e^6}$ D) $\frac{\pi}{2e^6}$ E) infinite

23. Evaluate $\int_1^\infty \frac{x^2}{\left(1+x^3\right)^2}dx$

A) –1/6 B) –1/24 C) 1/24 D) 1/6 E) nonexistent

24. Find $\int_0^\infty \frac{3x^2}{\left(1+x^3\right)^2}dx$

A) –1 B) 0 C) 1 D) 3 E) nonexistent

25. Evaluate $\int_0^3 \frac{3}{x}dx$

A) 0 B) 1 C) $3e$ D) e^3 E) nonexistent

26. (1985 BC5) Let f be the function defined by $f(x) = -\ln x$ for $0 < x \leq 1$ and let R be the region between the graph of f and the x-axis.

(a) Determine whether region R has finite area. Justify your answer.

(b) Determine whether the solid generated by revolving region R about the <u>y-axis</u> has finite volume. Justify your answer.

27. (1971 BC5) Determine whether or not $\int_0^\infty xe^{-x}\,dx$ converges. If it converges, give its value. Show your reasoning.

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Answer Key

L'Hopital's Rule

1.	D	1985	AB	#37	44%
2.	B	1988	AB	#23	73%
3.	C	1993	AB	#29	40%
4.	C	1985	BC	#23	25%
5.	D	1985	BC	#29	84%
6.	C	1993	BC	#24	58%
7.	C	1998	BC	#28	56%
8.	E	1993	BC	#42	25%
9.	C	1985	BC	#38	21%
10.	A	1988	BC	#35	50%
11.	C	2003	BC	#2	67%
12.	C	2008	BC	#3	73%
13.	D	Pearson BC1		#16	
14.	C	Pearson BC2		#28	
15.	A	Lucia BC1		#14	
16.	B	Lucia BC1		#44	
17.	B	Lucia BC2		#28	

Improper Integrals

18.	A	1988	BC	#7	82%
19.	E	1993	BC	#11	70%
20.	C	1998	BC	#25	45%
21.	C	2003	BC	#6	69%
22.	A	2008	BC	#11	52%
23.	D	Pearson BC1		#8	
24.	C	Pearson BC2		#25	
25.	E	Lucia BC1		#7	