AP Calculus Summer Packet Part 1: Algebra Review Name________________________

Perform the indicated operation.

1) $f(a) = a^2 + 3a$
$g(a) = 2a - 5$
Find $(f \circ g)\left(\frac{a}{3}\right)$

2) $g(n) = n + 2$
$h(n) = 2n - 5$
Find $g(h(-2n))$

3) $g(x) = 2x + 1$
$f(x) = 2x + 2$
Find $g(f(4x))$

4) $h(x) = -3x + 1$
$g(x) = -x^3 - 5$
Find $(h \circ g)(-3x)$

Factor each completely.

5) $8x^3 + 20x^2 + 10x + 25$

6) $5m^3 - 25m^2 + 4m - 20$

7) $5x^2 - 25x$

8) $m^2 - 8m + 12$

9) $n^3 - 6n^2 - 7n$

10) $21a^2 + 78a - 24$

11) $12b^2 + 18b - 84$

12) $5x^3 - 7x^2 + 2x$

13) $3x^3 - 19x^2 + 6x$

14) $25b^2 - 9$

15) $x^2 - 4$

16) $16p^2 - 8p + 1$

Condense each expression to a single logarithm.

17) $8\log 3 + 2\log 7$

18) $3\log_8 x - 3\log_8 y$

Evaluate each expression.

19) $\log_2 4$

20) $\log_6 \frac{1}{36}$

21) $\log_4 16$

22) $\log_5 125$

Write each expression in radical form.

23) $3^{\frac{4}{5}}$

24) $5^{\frac{4}{3}}$

25) $7^{\frac{5}{2}}$

26) $5^{\frac{7}{4}}$

Approximate the real zeros of each function to the nearest tenth. Use your calculator.

27) $f(x) = -x^4 + 2x^2 - x - 1$

28) $f(x) = x^3 - 4x^2 + 6$

29) $f(x) = x^4 - 4x^2 + 3x + 1$

30) $f(x) = x^3 - 2x^2 + 2$

Find the exact value of each trigonometric function.

31) $\cot \pi$

32) $\tan \pi$

33) $\csc -\frac{\pi}{4}$

34) $\cos \frac{5\pi}{6}$

35) $\cot \frac{\pi}{3}$

36) $\tan \frac{7\pi}{6}$

37) $\cot -\frac{4\pi}{3}$

38) $\cos \frac{4\pi}{3}$

Determine if each geometric series converges or diverges.

39) $5.9 + 2.95 + 1.475 + 0.7375...$

40) $-2.1 - 1.05 - 0.525 - 0.2625...$

41) $3 - 12 + 48 - 192...$

42) $5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27}...$

Find the sum of each infinite geometric series described.

43) $\sum_{n=1}^{\infty} -2 \cdot \left(\frac{2}{5}\right)^{n-1}$

44) $\sum_{n=1}^{\infty} 6 \cdot \left(\frac{1}{2}\right)^{n-1}$

45) $\sum_{k=1}^{\infty} -\frac{2}{3} \cdot \left(\frac{1}{2}\right)^{k-1}$

46) $\sum_{i=1}^{\infty} 5 \cdot \left(\frac{1}{3}\right)^{i-1}$

Answers to

1) $\frac{90 - 42a + 4a^2}{9}$
2) $-4n - 3$
3) $16x + 5$
4) $-81x^3 + 16$
5) $(4x^2 + 5)(2x + 5)$
6) $(5m^2 + 4)(m - 5)$
7) $5x(x - 5)$
8) $(m - 2)(m - 6)$
9) $n(n - 7)(n + 1)$
10) $3(7a - 2)(a + 4)$
11) $6(2b + 7)(b - 2)$
12) $x(5x - 2)(x - 1)$
13) $x(3x - 1)(x - 6)$
14) $(5b + 3)(5b - 3)$
15) $(x + 2)(x - 2)$
16) $(4p - 1)^2$
17) $\log (7^2 \cdot 3^8)$
18) $\log_8 \frac{x^3}{y^3}$
19) 2
20) −2
21) 2
22) 3
23) $(\sqrt[5]{3})^4$
24) $(\sqrt[3]{5})^4$
25) $(\sqrt{7})^5$
26) $(\sqrt[4]{5})^7$
27) Real Zeros: −1.5, −0.5
28) Real Zeros: −1.1, 1.6, 3.5
29) Real Zeros: −2.3, −0.3
30) Real Zeros: −0.8
31) Undefined
32) 0
33) $-\sqrt{2}$
34) $-\frac{\sqrt{3}}{2}$
35) $\frac{\sqrt{3}}{3}$
36) $\frac{\sqrt{3}}{3}$
37) $-\frac{\sqrt{3}}{3}$
38) $-\frac{1}{2}$
39) Converges
40) Converges
41) Diverges.
42) Converges
43) $-\frac{10}{3}$
44) 12
45) $-\frac{4}{3}$
46) $\frac{15}{2}$

AP Calculus Summer Packet Part 2: Pre-Calc Review   Name:________________________

I. Optimization:

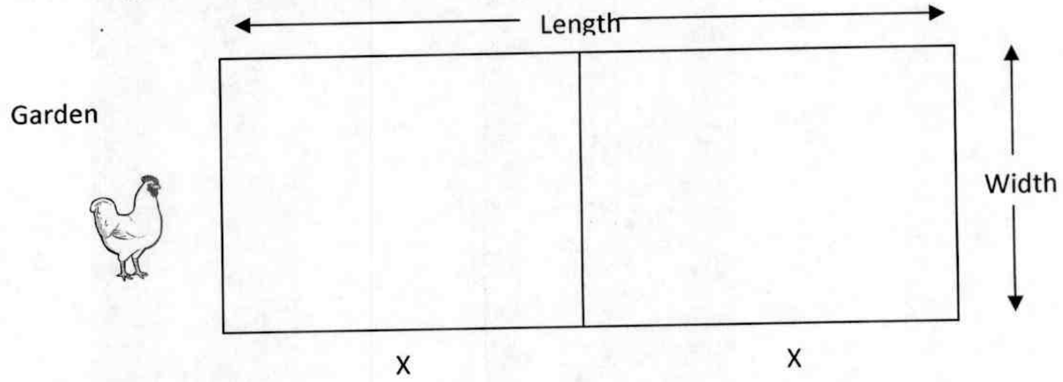
1) A farmer is building a rectangular pen for his cows. Using the side of the barn as one side of the pen, he will use 390 feet of fencing for the remaining three sides. He needs to know the dimensions of a pen with the largest possible area.

a) What are the dimensions of a pen with the largest possible area?

Length = ____________ Width = ____________

b) What is the maximum area?

2) Jamie is building a fenced in area for her garden and goats. She wants to keep them separated, so her garden stays safe. Her dad buys her 100 yards of fence to use. She will use some of the fence to make one large rectangle and the rest to divide that rectangle into two congruent parts.



a) What is the largest area that Jamie can design?

b) What are the length and the width of the largest area?

3) Mindy wants to build a <u>square</u> fenced in area for her chickens to roam around her backyard. She has determined that she needs at least 300 square feet of fenced in area for her chickens. Fencing costs $2 a foot.

a.) Write an equation for the area of the chicken pen. Make sure to define your variables.

b.) Write an equation for the cost of the fencing. Make sure to define your variables.

c.) What is minimum cost of the fencing in order to have at least 300 square feet? What are the dimensions of this pen?

d.) Mindy has determined that the type of fencing she purchased has limited her ability to build her chicken pen. Apparently, the length and width of her chicken pen has to have whole number values. What dimensions should Mindy make her pen so that she has at least 300 square feet of fenced in area but also minimizing the cost of the fencing? What is the cost using these new dimensions?

II. Trig Identities:

Simplify each expression to a single trig function or number.

1. $\sec\theta \sin\theta$

2. $\cos\theta \tan\theta$

3. $\tan^2\theta - \sec^2\theta$

4. $1 - \cos^2\theta$

5. $(1-\cos\theta)(1+\cos\theta)$

6. $(\sec x - 1)(\sec x + 1)$

7. $\dfrac{1}{\sin^2 A} - \dfrac{1}{\tan^2 A}$

8. $1 - \dfrac{\sin^2\theta}{\tan^2\theta}$

9. $\dfrac{1}{\cos^2\theta} - \dfrac{1}{\cot^2\theta}$

10. $\cos\theta(\sec\theta - \cos\theta)$

11. $\cos^2 A(\sec^2 A - 1)$

12. $(1-\cos x)(1+\sec x)(\cos x)$

III. Trig Equations: Solve for all $0 \le x < 2\pi$ **EXACT answers only!**

1. $\csc^2 x - 2\cot x = 0$

2. $2 = \sec x + \sec^2 x$

3. $\sqrt{2}\csc^2 x + \csc x = \sqrt{2}$

4. $\sec x + \cos x = \tan x \sin x - 1$

5. $\tan x - \sec x = \sqrt{3}$

6. $4\cos^4 x - 5\cos^2 x + 1 = 0$

7. $2\tan^2 x - 3\sec x + 3 = 0$

8. $\cot x - \sqrt{3} = \csc x$

9. $\tan^4 x - 2 = \tan^2 x + \sec^2 x$

10. $\cos x - \cot x = 0$

11. $\sec x = 1 + \tan x$

12. $\sin^2 x - \tan x \cos^2 x = 0$

13. $\sqrt{3}\tan x \sec x + 2\tan x = 0$

IV. Limits:

Evaluate each limit. Check your work using your graphing calculator.

1) $\lim_{x \to 1} (x - 2)$

2) $\lim_{s \to -2} -s$

3) $\lim_{t \to 1} 2t$

4) $\lim_{s \to 0} (s + 2)$

5) $\lim_{w \to -3} 2$

6) $\lim_{t \to 0} (2t + 2)$

7) $\lim_{r \to 0} (-r - 5)$

8) $\lim_{w \to 2} -2w$

9) $\lim_{w \to -1} (w + 1)$

10) $\lim_{r \to -2} (-r + 3)$

11) $\lim_{w \to -3} (-w^3 - 5w^2 - 3w + 2)$

12) $\lim_{x \to -1} -\sqrt[3]{2x + 2}$

13) $\lim_{x \to 3} (-x^3 + 10x^2 - 33x + 38)$

14) $\lim_{s \to 2} (-2s^2 + 8s - 8)$

15) $\lim_{s \to 0} \left(\frac{s^2}{2} + 3s + \frac{11}{2}\right)$

16) $\lim_{s \to 3} (-s^2 + 8s - 17)$

17) $\lim_{w \to -2} (-w^2 + 1)$

18) $\lim_{w \to 0} \left(-\frac{w^2}{2} + 3w - \frac{1}{2}\right)$

19) $\lim_{x \to 3} \sqrt{2x + 3}$

20) $\lim_{t \to -2} (2t^2 + 8t + 8)$

21) $\lim_{w \to \frac{3\pi}{4}} 2\cot(2w)$

22) $\lim_{t \to -1} \frac{15}{t^2 + 5}$

23) $\lim_{r \to \frac{2\pi}{3}} -2\cos(2r)$

24) $\lim_{t \to 2} \frac{t^2 - 2t - 15}{t - 5}$

25) $\lim_{t \to -1} \frac{t + 4}{t^2 + 14t + 49}$

26) $\lim_{w \to -3} \frac{w + 6}{w^2 + 8w + 16}$

27) $\lim_{x\to 3} \frac{x+3}{2x^2+2x+1}$

28) $\lim_{r\to -2} -\frac{25r}{r^2+25}$

29) $\lim_{s\to 0} \cos(s)$

30) $\lim_{w\to -2} \frac{w+4}{w^2+11w+28}$

Evaluate each limit.

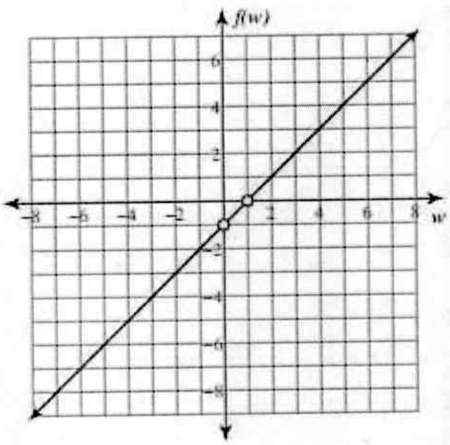
31) $\lim_{x\to 3} -\frac{x^2-3x}{x-3}$

32) $\lim_{t\to 2} -\frac{t-2}{t^2-4}$

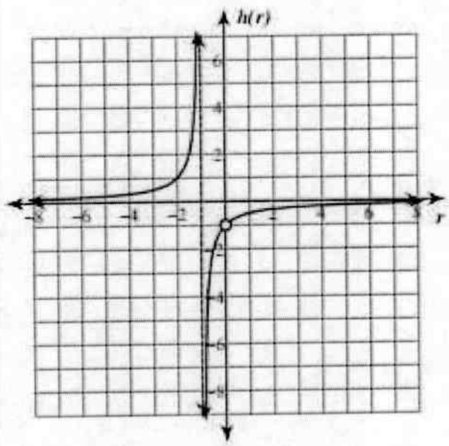
33) $\lim_{x\to -3} \frac{x^2+4x+3}{x+3}$

34) $\lim_{s\to 1} \frac{s-1}{s^2-1}$

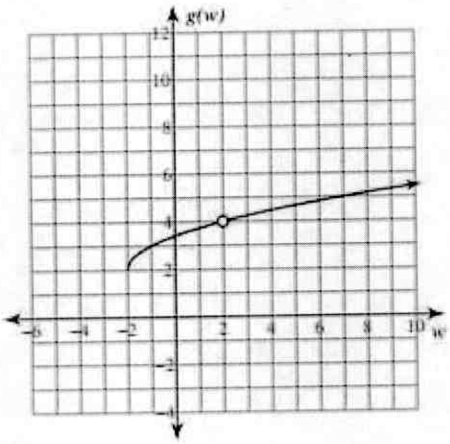
35) $\lim_{w\to 0} \frac{w}{\frac{1}{-1+w}+1}$



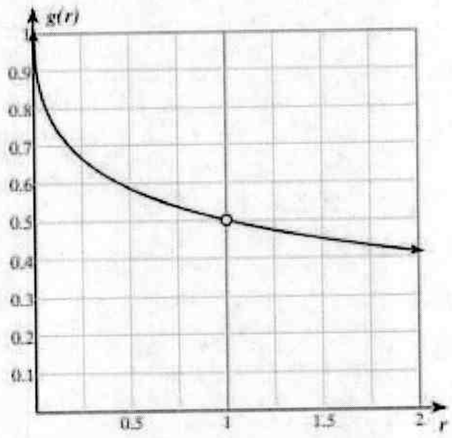
36) $\lim_{r\to 0} \frac{\frac{1}{1+r}-1}{r}$



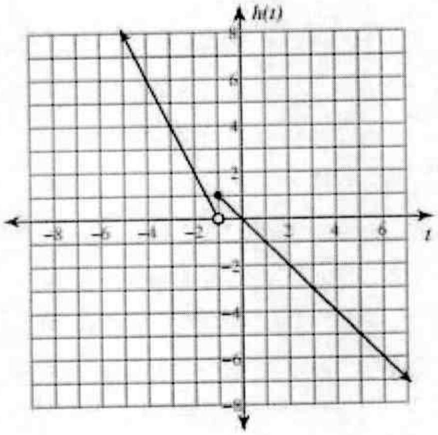
37) $\lim_{w \to 2} \dfrac{w-2}{\sqrt{w+2}-2}$



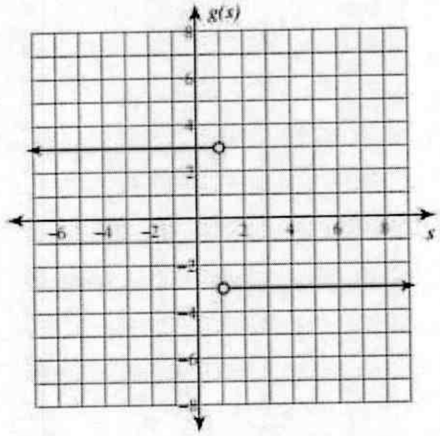
38) $\lim_{r \to 1} \dfrac{\sqrt{r}-1}{r-1}$



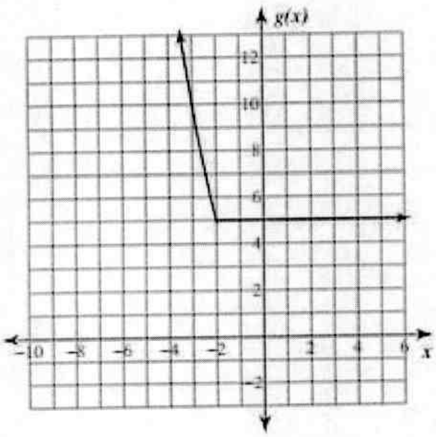
39) $\lim_{t\to -1} h(t),\ h(t) = \begin{cases} -2t - 2, & t < -1 \\ -t, & t \geq -1 \end{cases}$



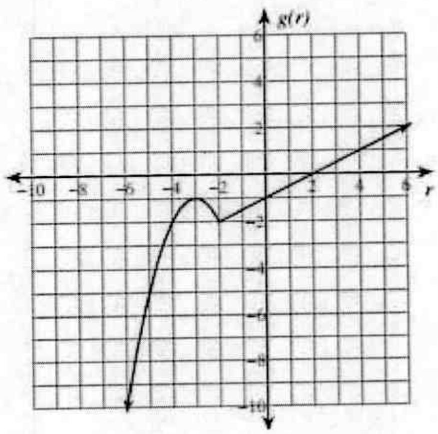
40) $\lim_{s\to 1} \dfrac{-3s + 3}{|-s + 1|}$



41) $\lim_{x\to -2} g(x),\ g(x) = \begin{cases} x^2 + 1, & x \leq -2 \\ 5, & x > -2 \end{cases}$



42) $\lim_{r\to -2} g(r),\ g(r) = \begin{cases} -r^2 - 6r - 10, & r \leq -2 \\ \dfrac{r}{2} - 1, & r > -2 \end{cases}$



KEY:

I.

1) length = 97.5 feet, width = 195 feet, 19012.5 square feet

2) A= 416. 7 yd^2 , $l = 25yd$ $w = \frac{50}{3}$ $or\ 16.7\ yd$

3)

a.) A = x^2 ; A is the area of the pen and x is the length/width
b.) C = 8x; C is the cost of the fencing
c.) $138.56; 17.32 feet
d.) 18 x 18 feet; $144

II. ANSWERS: 1. tanθ 2. sinθ 3. -1 4. $\sin^2\theta$ 5. $\sin^2\theta$ 6. $\tan^2\theta$ 7. 1 8. $\sin^2\theta$ 9. 1 10. $\sin^2\theta$ 11. $\sin^2 A$

12. $\sin^2 x$

III.

1.) $\frac{\pi}{4}, \frac{5\pi}{4}$ 2.) $0\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$ 3.) $\frac{5\pi}{4}, \frac{7\pi}{4}$ 4.) $\frac{2\pi}{3}, \frac{4\pi}{3}$

5.) $\frac{7\pi}{6}$ 6.) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, 0\pi, \pi$ 7.) 0π

8.) $\frac{4\pi}{3}$ 9.) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ 10.) $\frac{\pi}{2}, \frac{3\pi}{2}$ 11.) 0π

12.) $0\pi, \pi, \frac{\pi}{4}, \frac{5\pi}{4}$ 13.) $0\pi, \pi, \frac{5\pi}{6}, \frac{7\pi}{6}$

IV.

Answers to Limits Practice

1) -1
2) 2
3) 2
4) 2
5) 2
6) 2
7) -5
8) -4
9) 0
10) 5
11) -7
12) 0
13) 2
14) 0
15) $\frac{11}{2}$
16) -2
17) -3
18) $-\frac{1}{2}$
19) 3
20) 0
21) 0
22) $\frac{5}{2}$
23) 1
24) 5
25) $\frac{1}{12}$
26) 3
27) $\frac{6}{25}$
28) $\frac{50}{29}$
29) 1
30) $\frac{1}{5}$
31) -3
32) $-\frac{1}{4}$
33) -2
34) $\frac{1}{2}$
35) -1
36) -1
37) 4
38) $\frac{1}{2}$
39) Does not exist.
40) Does not exist.
41) 5
42) -2