

Integration Review

Calculus AB

Name: _____

Date: _____

I

A rectangular canal, 5m wide and 100m long has an uneven bottom. Depth measurements are taken at every 20m along the length of the canal. Use these depth measurements to construct a Riemann sum using right endpoints to estimate the volume of water in the canal.

| Distance | 0m | 20m | 40m | 60m | 80m | 100m |
|----------|------|------|------|------|------|------|
| Depth | 2.0m | 1.6m | 1.8m | 2.1m | 2.1m | 1.9m |

Suppose that f and g are continuous functions and that

$$\int_1^2 f(x) dx = -4$$

$$\int_1^5 f(x) dx = 6$$

$$\int_1^5 g(x) dx = 8$$

Find

a) $\int_2^2 f(x) dx$

c) $\int_1^2 3f(x) dx$

e) $\int_1^5 [f(x) + g(x)] dx$

b) $\int_1^5 g(x) dx$

d) $\int_2^5 f(x) dx$

f) $\int_1^5 [4f(x) - g(x)] dx$

I. Suppose that f and h are continuous functions such that

$$\int_1^9 f(x) dx = -1$$

$$\int_7^9 f(x) dx = 5$$

$$\int_7^9 h(x) dx = 4$$

Find

a) $\int_1^9 -2f(x) dx$

c) $\int_7^9 [2f(x) - h(x)] dx$

e) $\int_1^7 f(x) dx$

b) $\int_7^9 [f(x) + h(x)] dx$

d) $\int_9^1 f(x) dx$

f) $\int_9^7 [h(x) - f(x)] dx$

II. Evaluate each integral

a) $\int_3^1 7 dx$

c) $\int_2^3 \frac{x^2 + 2x^3}{x^2} dx$

e) $\int_0^\pi (e^x + \cos x) dx$

b) $\int_0^9 5\sqrt{x} dx$

d) $\int_0^{\sqrt{2}} (t - \sqrt{2}) dt$

f) $\int_0^4 \sqrt{x}(x+1) dx$

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III. Find dy/dx of the following

a. $y = \int_0^x \sqrt{1+t^2} dt$

b. $y = \int_0^{\sqrt{x}} \sin(t^2) dt$

c. $y = \int_{x^2}^0 \cos t dt$

IV.

Consider the integral $\int_0^4 16 - t^2 dt$. Find the Riemann sum for this integral using:

- right-hand sums for $n = 4$.
- left-hand sums for $n = 4$.
- Sketch a graph for (a)-(b). Which underestimates the actual value of the integral? Overestimates? What can you say in general?
- Now suppose that $v(t) = 16 - t^2$ is the velocity (in ft/s) of a car after t seconds, so the car comes to a stop after 4 seconds. Suppose that a kitten is 45 feet in front of the car at $t = 0$. Based on (a)-(c), can we determine whether or not the car struck the kitten?
- Compute the actual value of the integral to find that the kitten lives to see another day.

Use the table of values to estimate

$$\int_0^6 f(x) dx.$$

Use three equal subintervals and the (a) left endpoints, (b) right endpoints, and (c) midpoints. If f is an increasing function, how does each estimate compare with the actual value? Explain your reasoning.

| | | | | | | | |
|--------|----|---|---|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$ | -6 | 0 | 8 | 18 | 30 | 50 | 80 |

For each problem, find the values of c that satisfy the Mean Value Theorem for Integrals.

3) $f(x) = -\frac{x^2}{2} + x + \frac{3}{2}$; $[-3, 1]$

4) $f(x) = \frac{4}{x^2}$; $[-4, -2]$

For each problem, find the average value of the function over the given interval.

5) $f(x) = -x^3 + 7x^2 - 11x + 3$; $[1, 5]$

6) $f(x) = -x^5 + 3x^3$; $[0, 1]$