AP Calculus BC

Alternating Series Error

*check terms dec in abs Value

Name: Key

Alternating Series Remainder

Suppose an alternating series satisfies the conditions of the Alternating Series Test: namely, that $\lim_{n\to\infty} a_n = 0$ and $\{a_n\}$ is a decreasing sequence ($a_{n+1} < a_n$). If the series has a sum S, then Remainder = $|R_n| = |S - S_n| \le a_{n+1}$, where S_n is the nth partial sum of the series.

In other words, if an alternating series satisfies the conditions of the Alternating Series Test, you can approximate the sum of the series by using the nth partial sum, S_n, and your error will have an absolute value no greater than the first term left off, a_{n+1}

1) Given the following series: $\sum_{n=1}^{10} (-1)^{n+1} \frac{1}{n^n}$

$\frac{1}{(n+1)^{n+1}} < \frac{1}{n^n}$

$n^n < (n+1)^{n+1}$

a. Find the sum using your calculator. .78343

b. What is the maximum value of the error if the sum $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^n}$ is approximated by the answer to part (a)?

next term
n=1

$\frac{1}{11^{11}} = 3.505 \times 10^{-12}$

c. If the sum of the first twenty terms was used instead in part (a), would the error in part (b) be increased or decreased? Explain your answer.

terms ↑ , error ↓

2) Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

5th term

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(0.1)^n}{n}$$

$\frac{(.1)^{n+1}}{n+1} < \frac{(.1)^n}{n}$

$n(.1)^{n+1} < (n+1)(.1)^n$

$\frac{(0.1)^5}{5} = 2 \times 10^{-6}$

3) Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

5th term

$\frac{1}{2^{n+1}} < \frac{1}{2^n}$

$2^n < 2^{n+1}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n}$$

$\frac{1}{2^5} = \frac{1}{32}$

4) Find the error for estimating sin (0.5) by the third-degree Taylor polynomial for sin x.

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$|\text{Error}| \le \frac{(.5)^5}{5!}$

5) The error in estimating e^{-2} using five terms of the Taylor series for e^{-x} is not greater than ___________? $x = 2$

$e^x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right) + \frac{x^5}{5!}$

$e^{-x} = \left(1 - x + \frac{(-x^2)}{2!} + \frac{(-x)^3}{3!} + \frac{(-x)^4}{4!}\right) + \frac{(-x)^5}{5!}$

$\frac{2^5}{5!}$

$5! = 120$
$6! = 720$
$7! = 5040$

6) Given $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$

a) Find the upper bound for error if the first 4 terms are used for S_n.

$\frac{1}{5!} = \frac{1}{120}$

b) What is the smallest value of n for which S_n approximates the sum within 0.005?

$\frac{1}{(n+1)!} < .005$ $\quad \frac{1}{(n+1)!} < \frac{5}{1000}$

$1000 < 5(n+1)!$
$200 < (n+1)!$
$n = 5$

7) Given $\sum_{n=1}^{\infty}\left(\frac{-1}{5}\right)^n$

a) Find the upper bound for error if the first 4 terms are used for S_n.

$\left|\left(\frac{-1}{5}\right)^5\right| = \left(\frac{1}{5}\right)^5 = \frac{1}{3125}$

b) What is the smallest value of n for which S_n approximates the sum within 0.005?

$\left(\frac{1}{5}\right)^{n+1} < .005$

$\left(\frac{1}{5}\right)^n\left(\frac{1}{5}\right) < \frac{5}{1000}$

$\left(\frac{1}{5}\right)^n < \frac{25}{1000}$

$\frac{1}{5^n} < \frac{25}{1000}$

$1000 < 25 \cdot 5^n$
$40 < 5^n$
$n = 3$

8) Given $\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{n}\right)$

a) Find the upper bound for error if the first 10 terms are used for S_n.

$R \le \frac{1}{11}$

b) Find the upper bound for error if the first 99 terms are used for S_n.

$R \le \frac{1}{100}$

2012 BC 6:

The function g has derivatives of all orders, and the Maclaurin series for g is

$$\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+3} = \frac{x}{3} - \frac{x^3}{5} + \frac{x^5}{7} - \cdots.$$

(a) Using the ratio test, determine the interval of convergence of the Maclaurin series for g.

(b) The Maclaurin series for g evaluated at $x = \frac{1}{2}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $g\left(\frac{1}{2}\right)$ using the first two nonzero terms of this series is $\frac{17}{120}$. Show that this approximation differs from $g\left(\frac{1}{2}\right)$ by less than $\frac{1}{200}$.

(c) Write the first three nonzero terms and the general term of the Maclaurin series for $g'(x)$.

2012 BC6:

a) $\lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{2(n+1)+3} \cdot \frac{2n+3}{x^{2n+1}} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{x^{2n}} \cdot x^3}{2n+5} \cdot \frac{2n+3}{\cancel{x^{2n}} \cdot x} \right|$

$\lim_{n\to\infty} \left| \frac{2n+3}{2n+5} \cdot x^2 \right|$

which x-values can you square to make true?

$|x^2| < 1$

$-1 < x^2 < 1$

$\sqrt{0} < \sqrt{x^2} < \sqrt{1}$

$0 < |x| < 1$

$-1 < x < 1$

Endpts:

$x=-1$: $\frac{(-1)^n(-1)^{2n+1}}{2n+3}$

$\frac{(-1)^n \cancel{(-1)^{2n}}(-1)}{2n+3} = \frac{-1(-1)^n}{2n+3}$

AST conv

$x=1$:

$\frac{(-1)^n(1)^{2n+1}}{2n+3}$ AST conv

IOC: $-1 \le x \le 1$

b) $\left| g\left(\frac{1}{2}\right) - \frac{17}{120} \right| < \frac{\left(\frac{1}{2}\right)^5}{7} = \frac{1}{224} < \frac{1}{200}$

3rd term

c) $g'(x) = \frac{1}{3} - \frac{3}{5}x^2 + \frac{5}{7}x^4$

$(-1)^n \left(\frac{2n+1}{2n+3}\right) \cdot x^{2n}$