Alternating Series Error Practice

1. How many terms of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$ are needed to approximate the sum of the series with error < 0.001?
   a. 3
   b. 4
   c. 5
   d. 6
   e. 7

2. Given the alternating series $\sum_{n=0}^{\infty}(-0.25)^n$
   Find the error when the series is approximated by the first four terms of the series.

3. Find an upper bound for the error when the sum is estimated using the first ten terms of the following series: $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$

4. What is the least positive integer n that will make the error estimate of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^3}$ less than 0.001?

5. Given $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$
   a) Find the upper bound for error if the first 4 terms are used for S_n.
   b) What is the smallest value of n for which S_n approximates the sum within 0.005?

X Given $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n^3}\right)$, what is the least positive integer, n, for which the error is less than .001?

6. Find the 4th degree Taylor polynomial for cosx about x =0.
   a. Use your polynomial to approximate the value of cos 0.8.
   b. Find the maximum error possible.

7. The maximum error incurred by approximating the sum of the series $1-\frac{1}{2!}+\frac{2}{3!}-\frac{3}{4!}+\frac{4}{5!}-\ldots$ by the sum of the first six terms is

a. 0.001190 b. 006944 c. 0.33333 d. 0.125000 e. None of these

8 **1982 BC5**

(a) Write the Taylor series expansion about $x=0$ for $f(x)=\ln(1+x)$. Include an expression for the general term.

(b) For what values of x does the series in part (a) converge?

(c) Estimate the error in evaluating $\ln\left(\frac{3}{2}\right)$ by using only the first five nonzero terms of the series in part (a). Justify your answer.

9. **1984 BC4**

Let f be the function defined by $f(x)=\sum_{n=1}^{\infty}\frac{x^n n^n}{3^n n!}$ for all values of x for which the series converges.

(a) Find the radius of convergence of this series.

(b) Use the first three terms of this series to find an approximation of $f(-1)$.

(c) Estimate the amount of error involved in the approximation in part (b). Justify your answer.

Alternating Series Error Practice:

1) $\frac{1}{(n+1)^4} < .001$

C

$\frac{1}{(n+1)^4} < \frac{1}{1000}$

$1000 < (n+1)^4$

$6^4 = 1296$

$5^4 = 625$

$n+1 = 6$

$n = 5$

2) 5^{th} term: $n=4$

$(-0.25)^4 = \frac{1}{256}$

3) $\frac{1}{11}$

5) a) $R_4 \leq \frac{1}{(n+1)^2} = \frac{1}{5^2} = \frac{1}{25}$

b) $\frac{1}{(n+1)^2} < .005$

$\frac{1}{(n+1)^2} < \frac{5}{1000}$

$1000 < 5(n+1)^2$

$200 < (n+1)^2$ $\quad 15^2 = 225$

$n = 14$

4) $\frac{1}{(n+1)^3} < .001$

$\frac{1}{(n+1)^3} < \frac{1}{1000}$

$1000 < (n+1)^3$

$n = 10$

6) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

a) $\cos(.8) = 1 - \frac{(.8)^2}{2!} + \frac{(.8)^4}{4!} = .6971$

b) Error $< \frac{(.8)^6}{6!} = 3.641 \times 10^{-4}$

7) 6^{th} term: $\frac{5}{6!}$

Error is next term

$\frac{6}{7!}$

(A)

1982 BC5 :

a) $f(x) = \ln(1+x)$ $\quad f(0) = 0$

$f'(x) = (1+x)^{-1}$ $\quad f'(0) = 1$

$f''(x) = -(1+x)^{-2}$ $\quad f''(0) = -1$

$f'''(x) = 2(1+x)^{-3}$ $\quad f'''(0) = 2$

$$f(x) = 0 + \frac{1x^1}{1!} - \frac{1x^2}{2!} + \frac{2x^3}{3!}$$

or

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \quad \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{x^n}{n}$$

b) $$\lim_{n\to\infty} \left| \frac{(-1)^{n+2} \cdot x^{n+1}}{n+1} \cdot \frac{n}{(-1)^{n+1} \cdot x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^n}\cancel{(-1)^2} \cdot \cancel{x^n} \cdot x}{n+1} \cdot \frac{n}{\cancel{(-1)^n}(-1) \cdot \cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \left| \cancel{\frac{n}{n+1}}_{1} \cdot -x \right|$$

$|-x| < 1$

$-1 < -x < 1$

$1 > x > -1$

Endpts:

$x = -1$: $\frac{(-1)^{n+1}(-1)^n}{n} = \frac{(-1)^n(-1)(-1)^n}{n} = \frac{-1}{n}$ div n^{th} term

$x = 1$: $\frac{(-1)^{n+1}(1)^n}{n} = \frac{(-1)^n(-1)(1)^n}{n} = \frac{-(-1)^n}{n}$ conv AST

c) 6^{th} term: $\frac{x^6}{6}$

Error $\leq \frac{(\frac{1}{2})^6}{6} = \frac{1}{384} \approx .0026$

$1 + x = \frac{3}{2}$

$x = \frac{1}{2}$

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1984 BC4 :

a) $\lim_{n\to\infty} \left| \frac{x^{n+1}(n+1)^{n+1}}{3^{n+1}(n+1)!} \cdot \frac{3^n \cdot n!}{x^n \cdot n^n} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x\,(n+1)^n \cancel{(n+1)}}{\cancel{3^n} \cdot 3\,\cancel{(n+1)} \cdot \cancel{n!}} \cdot \frac{\cancel{3^n} \cdot \cancel{n!}}{\cancel{x^n} \cdot n^n} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)^n}{n^n} \cdot \frac{x}{3} \right| \longrightarrow \left| e \cdot \frac{x}{3} \right| < 1$

$\lim_{n\to\infty} \left(\frac{n+1}{n} \right)^n$

$-1 < \frac{xe}{3} < 1$

$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$

$-3 < xe < 3$

$-\frac{3}{e} < x < \frac{3}{e}$

ROC: $\frac{3}{e}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n^n}{3^n \cdot n!}$

$f(-1) \approx -\frac{1}{3} + \frac{2}{9} - \frac{1}{6} \approx -\frac{5}{18}$

c) Error $< \frac{(-1)^4 \cdot 4^4}{3^4 \cdot 4!} = \frac{32}{243}$