AB Practice Exam

No calculator is allowed for these questions.

1. If $xy - y = 2x + 4$, $\frac{dy}{dx}$ is

(A) $\frac{y-2}{x-1}$

(B) $\frac{2-y}{x-1}$

(C) $\frac{y-6}{x-1}$

(D) $\frac{2}{x-1}$

(E) $\frac{y-2}{x+1}$

2. Let $f(x)$ be an odd function and $g(x)$ be even. Which of the following statements are true?

I $\int_{-2}^{2} f(x)\,dx = 0$

II $\int_{-2}^{2} g(x)\,dx = 0$

III $\int_{-2}^{2} f(x)g(x)\,dx = 0$

(A) II

(B) III

(C) I and II

(D) I and III

(E) I, II, and III

3. $\lim_{x \to 0} \frac{\sin 3x}{5x} =$

(A) $\frac{3}{5}$

(B) $\frac{5}{3}$

(C) 3

(D) 5

(E) no limit exists

4. For which of the following x-values on the graph of $y = 2x - x^2$ is $\frac{dy}{dx}$ the largest?

(A) -2.7

(B) -2.2

(C) 0

(D) 1

(E) 2.7

5. If $y = e^{2x} + \tan 2x$, then $y'(\pi) =$

(A) $2e^{2\pi}$

(B) $e^{2\pi} + 1$

(C) $2e^{2\pi} + 2$

(D) $2e^{\pi} - 2$

(E) 0

6. Write the equation of the line tangent to $y = e^{x+1}$ at $x = 0$.

(A) $y = ex + e$

(B) $y = x$

(C) $y = x + 1$

(D) $y = x + e$

(E) $y = ex + 1$

7. If the acceleration of a particle is given by $a(t) = 2e^t$ and at $t = 1$ the velocity is 2, then $v(0)$ is

(A) 0

(B) $2 - 2e$

(C) $2 - \frac{1}{2}e$

(D) $4 - 2e$

(E) 2

8. If $3xy + 2y^2 = 5$, find $\frac{dy}{dx}$ at (1, 1).

(A) $-\frac{3}{7}$

(B) 0

(C) $\frac{3}{7}$

(D) 7

(E) undefined

9. The graph of $y = \frac{\ln(1-x)}{x+1}$ has vertical asymptote(s) at

(A) $x = 1$

(B) $x = 0$

(C) $x = \pm 1$

(D) $x = -1$

(E) no vertical asymptote

10. $\frac{d}{dt}\int_t^{t^2} \frac{1}{x}\,dx =$

(A) $\frac{1}{t^2}$

(B) $\ln(t)$

(C) $\ln(t^2)$

(D) $\frac{1}{t}$

(E) $\ln\frac{1}{t}$

11. $\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{4} + h\right) - \cos\frac{\pi}{4}}{h} =$

(A) $-\frac{\sqrt{3}}{2}$

(B) $-\frac{\sqrt{2}}{2}$

(C) 0

(D) $\frac{\sqrt{2}}{2}$

(E) 1

12. $\int (3x^2 - \cos x)\,dx =$

(A) $6x + \sin(x) + C$

(B) $x^3 + \sin(x) + C$

(C) $x^3 - \sin(x) + C$

(D) $6x - \sin(x) + C$

(E) $3x^3 - \sin(x) + C$

13. Find the area under the curve $y = x^2 + 1$ on the interval [1, 2].

(A) $\frac{7}{3}$

(B) 3

(C) $\frac{10}{3}$

(D) 4

(E) 7

14. $\int_{-1}^{2} \frac{x}{x^2+1}\,dx =$

(A) $\ln\frac{2}{5}$

(B) 0

(C) $\frac{1}{2}\ln\frac{5}{2}$

(D) $\frac{1}{2}\ln 3$

(E) undefined

15. $\int_0^1 e^{3x+2}\,dx =$

(A) $\frac{1}{3}e^5$

(B) $\frac{1}{3}(e^5 - e^2)$

(C) $\frac{1}{3}(e^5 - 1)$

(D) $e^5 - 1$

(E) $e^5 - e^2$

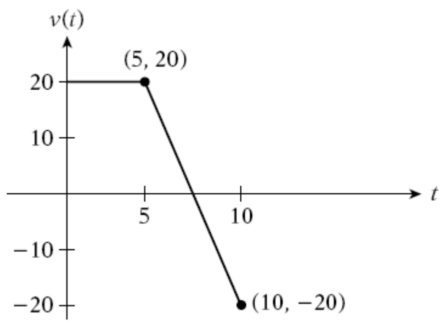
16. If $y = \sin^2 5x$, $\frac{dy}{dx} =$

(A) $5 \sin 10x$
(B) $5 \cos 5x$
(C) $5 \sin 5x$
(D) $10 \sin 10x$
(E) $10 \sin^2 5x$

17. For what values of x is $f(x) = 2x^3 - x^2 + 2x$ concave up?

(A) $x < \frac{1}{6}$
(B) $x < 0$
(C) $x > 0$
(D) $x > \frac{1}{6}$
(E) $x > 6$

Questions 18 and 19 refer to the graph of the velocity v(t) *of an object at time* t *shown below.*



18. If $x(t)$ is the position of the object at time t, and $a(t)$ is the acceleration of the object at time t, which of the following is true?

(A) $v(5) > v(2)$
(B) $x(5) > x(2)$
(C) $a(6) > a(2)$
(D) $x(10) < x(5)$
(E) $a(9) > a(6)$

19. $\int_0^{10} v(t)\, dt =$

(A) 0
(B) 5
(C) 50
(D) 75
(E) 100

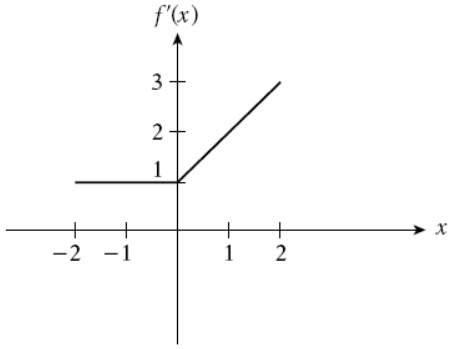
20. If $f(1) = 2$ and $f'(1) = 5$, use the equation of the line tangent to the graph of f at $x = 1$ to approximate $f(1.2)$.

(A) 1
(B) 1.2
(C) 3
(D) 5.4
(E) 9

21. The equation of the line tangent to $y = \tan^2(3x)$ at $x = \frac{\pi}{4}$ is

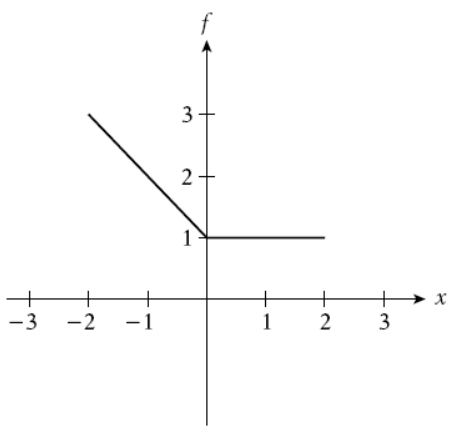
(A) $y = -12x + 3\pi - 1$
(B) $y = -12x + 3\pi + 1$
(C) $y = -6x + \frac{3\pi}{2} + 1$
(D) $y = -12x + 3\pi + 3$
(E) $y = -12x + \pi + 3$

22. The graph of $f'(x)$ is shown below.

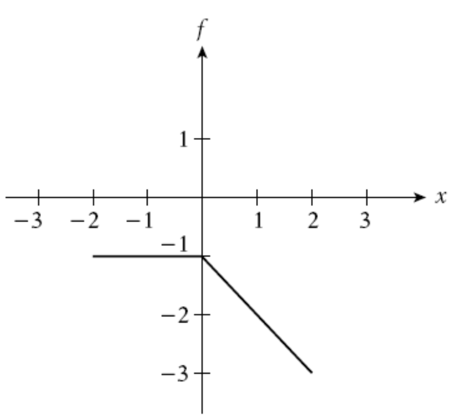


Which of the following could be the graph of f?

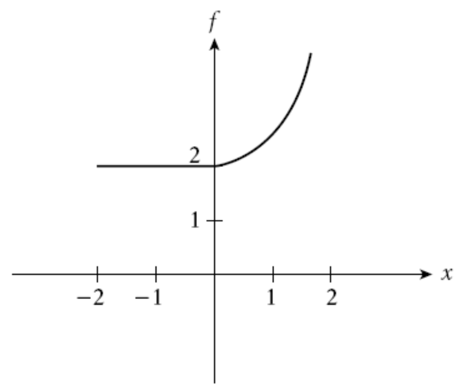
(A)



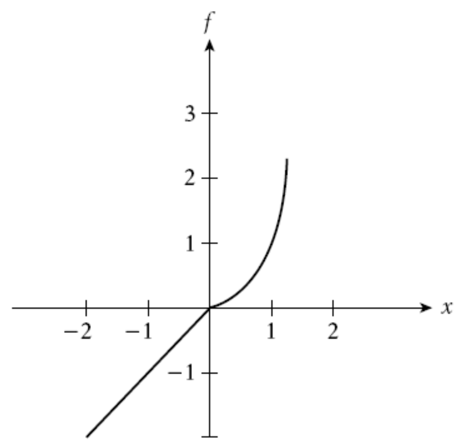
(B)



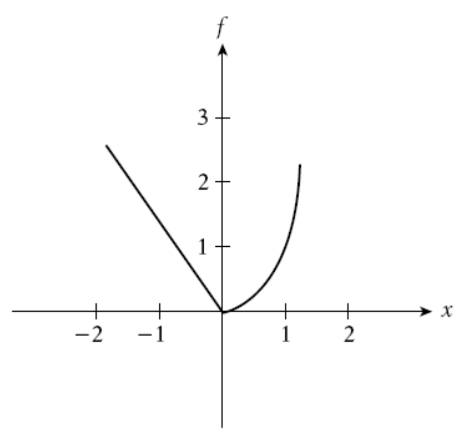
(C)



(D)



(E)



23. At what value of x is the line tangent to the graph of $y = x^2 + 3x + 5$ perpendicular to the line $x - 2y = 5$?

(A) $-\frac{5}{2}$

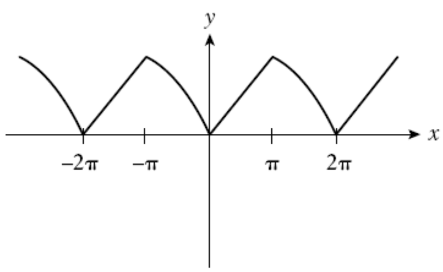
(B) -2

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

(E) $\frac{5}{2}$

24. The function $f(x)$ graphed here is called a sawtooth wave. Which of the following statements about this function is true?



(A) $f(x)$ is continuous everywhere.

(B) $f(x)$ is differentiable everywhere.

(C) $f(x)$ is continuous everywhere but $x = n\pi$

(D) $f(x)$ is is an even function.

(E) $f(x)$ is a one-to-one function.

25. Find the derivative of $y = x^2e^{x^2}$.

(A) $2xe^{x^2}(x^2 + 1)$

(B) $2xe^{x^2}$

(C) $2x^3e^{x^2}$

(D) $4x^2e^{x^2}$

(E) $x^4e^{x^2}$

26. $\int_0^{\frac{\pi}{2}} e^{2-\cos x} \sin x \, dx =$

(A) $e^2 - e$

(B) 1

(C) 0

(D) e^2

(E) does not exist

27. The average value of $f(x) = -\frac{1}{x^2}$ on $\left[\frac{1}{2}, 1\right]$ is

(A) -2

(B) -1

(C) $-\frac{1}{2}$

(D) 2

(E) undefined

28. $f(x) = |x^2 - 3x|$. Find $f'(1)$.

(A) -3

(B) -1

(C) 1

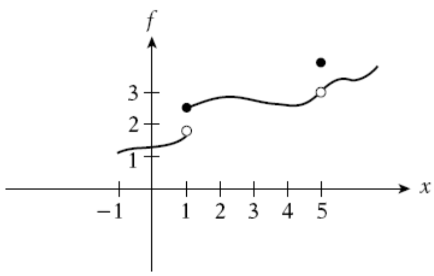
(D) 3

(E) does not exist

Part B

A graphing calculator is required for some questions.

1. Which of the following statements is true about the figure?



(A) $\lim_{x \to 5} f(x)$ exists

(B) $\lim_{x \to 1} f(x)$ exists

(C) $\lim_{x \to 5} f(x) = f(5)$

(D) $\lim_{x \to 1} f(x) = f(1)$

(E) $\dfrac{f(5) - f(1)}{5 - 1} = f'(c)$

2. How many points of inflection are there for the function $y = x + \cos 2x$ on the interval $[0, \pi]$?

(A) 0

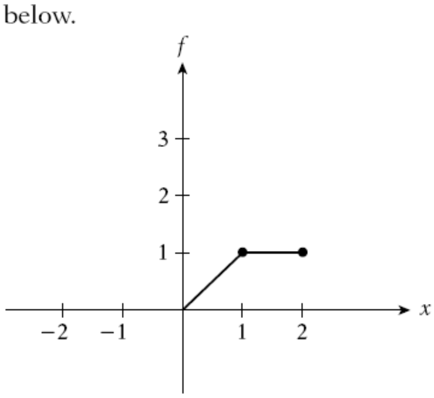
(B) 1

(C) 2

(D) 3

(E) 4

3. The graph of the function $f(x)$ is shown below.



If $F'(x) = f(x)$, and $F(0) = -3$, then $F(2) =$

(A) -4.5

(B) -1.5

(C) 1.5

(D) 3

(E) 4.5

4. If $\lim_{h \to 0} \dfrac{f(3 + h) - f(3)}{h} = 0$, then which of the following must be true?

I f has a derivative at $x = 3$.
II f is continuous at $x = 3$.
III f has a critical value at $x = 3$.

(A) I only

(B) II only

(C) I and II

(D) I and III

(E) I, II, and III

5. Consider the function $y = x^3 - x^2 - 1$. For what value(s) of x is the slope of the tangent equal to 5?

(A) -1 only

(B) $\frac{5}{3}$ only

(C) -1 and $\frac{5}{3}$

(D) $\frac{1}{3}$

(E) 2.219

6. A pebble thrown into a pond creates circular ripples such that the rate of change of the circumference is 12π cm/sec. How fast is the area of the ripple changing when the radius is 3 cm?

(A) 6π cm^2/sec

(B) 2π cm^2/sec

(C) 12π cm^2/sec

(D) 36π cm^2/sec

(E) 6 cm^2/sec

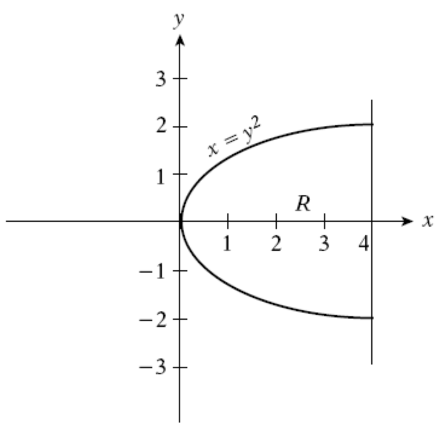
7. If $y = x^2 + 1$, what is the smallest positive value of x such that $\sin y$ is a relative maximum?

(A) 0.756
(B) 0.841
(C) 1
(D) 1.463
(E) 1.927

8. Find the area in the first quadrant bounded by $y = 2 \cos x$, $y = 3 \tan x$, and the y-axis.

(A) 0.347
(B) 0.374
(C) 0.432
(D) 0.568
(E) 1.040

9. In the following figure, region R bounded by $x = y^2$ and the line $x = 4$ is the base of a solid. Cross sections of the solid perpendicular to the x-axis are semicircles with diameters in the plane of the region.



Which of the following represents the volume of the solid?

(A) π
(B) 4π
(C) 8π
(D) $\frac{32}{3}\pi$
(E) 16π

10. $f'(x) = x^3(x - 2)^4(x - 3)^2$. $f(x)$ has a relative maximum at $x =$

(A) 0
(B) 2
(C) 2 and 3
(D) 0 and 3
(E) There is no relative maximum.

11. Find the average rate of change of $f(x) = \sec x$ on the interval $\left[0, \frac{\pi}{3}\right]$.

(A) 0.396
(B) 0.955
(C) 1.350
(D) 1.910
(E) undefined

12. $a(t) = \dfrac{5t^2 + 1}{5t}$ and $v(1) = 1$. Find $v(2)$.

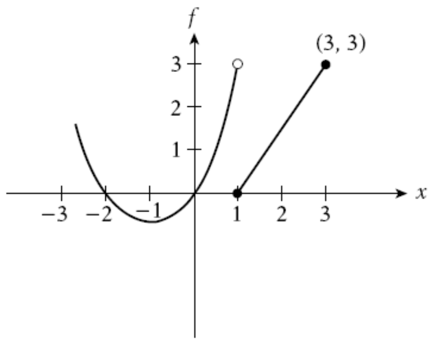
(A) 1.139
(B) 2.10
(C) 2.139
(D) 2.639
(E) undefined

13. Use the table shown to approximate the area under the curve of $y = f(x)$ using trapezoids.

x	**y**
0	1
1	2
3	4
4	1

(A) 5.5
(B) 8
(C) 10
(D) 11
(E) 20

14. In the figure shown, which of the following is true?



(A) $\lim_{x \to 1} f(x) = 3$

(B) $\lim_{x \to 1+} f(x) = 3$

(C) $f'(1) = 1$

(D) $f(1) = 3$

(E) The average rate of change of $f(x)$ on $[1, 3]$ equals $f'(2)$.

15. The region enclosed by the graphs of $y = \sqrt{x}$, $y = 2$, and the y-axis is rotated about the line $y = 4$. Write an integral that represents the volume of the solid generated.

(A) $2\pi \int_0^2 \sqrt{x}\, dx$

(B) $\pi \int_0^2 \left(4 - \sqrt{x}\right) dx$

(C) $\pi \int_0^4 \left(\left(4 - \sqrt{x}\right)^2 - 4\right) dx$

(D) $\pi \int_0^2 \left(\left(4 - \sqrt{x}\right)^2 - 4\right) dx$

(E) $\pi \int_0^4 \left(4 - \sqrt{x}\right)^2 dx$

16. The position of a particle on a line is given by $x(t) = t^3 - t$, $t \geq 0$. Find the distance traveled by the particle in the first two seconds.

(A) 0.385

(B) 3.385

(C) 6

(D) 6.385

(E) 6.770

17. $\int_a^b |f(x)|\, dx = p$ and $\left|\int_a^b f(x)\, dx\right| = q$. Which of the following must be true?

(A) $p = q$

(B) $p \geq q$

(C) $p \leq q$

(D) $p > q$

(E) $p < q$

Part A

A graphing calculator is required for some questions.

1. A rocket is launched with an initial velocity of zero, and with acceleration in feet per second per second defined by

$$a(t) = \begin{cases} 20e^{-t/2}, & \text{for } 0 \le t \le 10 \text{ seconds} \\ -16, & \text{for } t > 10 \text{ seconds} \end{cases}.$$

(a) At what time does the rocket begin to descend?

(b) How high does the rocket reach?

(c) What is the velocity when the rocket impacts the earth?

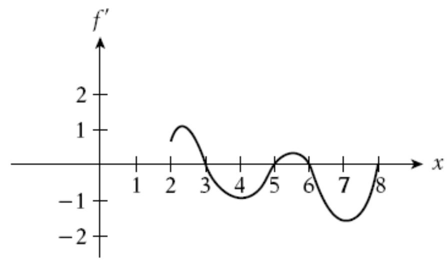
(d) Write a formula for the position of the rocket with respect to time for $t > 10$ seconds.

2. A leaky cylindrical oilcan has a diameter of 4 inches and a height of 6 inches. The can is full of oil and is leaking at the rate of 2 in.^{3}/hr. The oil leaks into an empty conical cup with a diameter of 8 inches and a height of 8 inches.

(a) At what rate is the depth of the oil in the conical cup rising when the oil in the cup is 3 inches deep?

(b) When the oilcan is empty, what is the depth of the oil in the conical cup?

3. Consider the graph of the derivative of f.



(a) At what value(s) of x does f have a relative maximum? Justify your answer.

(b) On what intervals is f concave up?

(c) At what value(s) of x does f have a point of inflection? Justify your answer.

(d) Is $f(3) > f(2)$? Justify your answer.

Part B

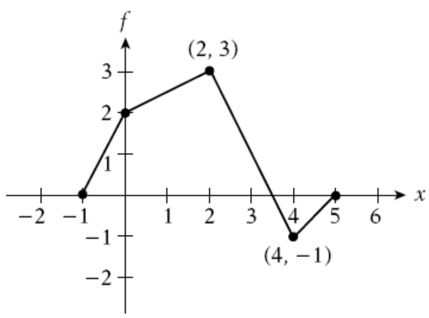
No calculator is allowed for these questions.

4. Region R is bounded by the graph of $y = x^{2/3}$, the x-axis, and the line $x = 8$.

(a) For what value of k, $0 < k < 8$, does the line $x = k$ divide region R into two parts equal in area?

(b) Region R is rotated about the x-axis. Find the value of p if the lines $x = p$ and $x = q$ $(p < q)$ divide the solid into three parts that have the same volume.

5. Consider the graph of $f(x)$ shown, which consists of four straight line segments.



If $h(x) = \int_1^x f(t)\,dt$,

(a) Find $h(2)$ and $h(-1)$.

(b) On what interval(s) is $h(x)$ decreasing? Justify your answer.

(c) What are the critical values of h? Justify your answer.

(d) What are the points of inflection of h? Justify your answer.

(e) Find the absolute maximum and the absolute minimum of h. Justify your answer.

6. (a) Draw a slope field for the differential equation

$$\frac{dy}{dx} = x(y - 1)$$

for $0 \le x \le 2$ and $0 \le y \le 3$.

(b) On the slope field drawn in part (a), sketch a solution to the differential equation with initial condition $y(0) = 2$.

(c) Solve the differential equation with initial condition $y(0) = -1$.

AB Model Examination 1

Section I
Part A (pages 283–287)

1. (B) $\dfrac{2-y}{x-1}$

$$x \cdot \frac{dy}{dx} + y \cdot 1 - \frac{dy}{dx} = 2$$

$$x \cdot \frac{dy}{dx} - \frac{dy}{dx} = 2 - y$$

$$\frac{dy}{dx}(x-1) = 2 - y$$

$$\frac{dy}{dx} = \frac{2-y}{x-1}$$

2. (C) I and III

3. (A) $\frac{3}{5}$

4. (A) -2.7

$\frac{dy}{dx} = 2 - 2x$; therefore, the lowest x-value will yield the highest value when substituted for x.

5. (C) $2e^{2\pi} + 2$

$$y' = 2 \cdot e^{2x} + \frac{2}{\cos^2(2x)}$$

6. (A) $y = ex + e$

$$y' = e^{x+1}$$

Substitute 0 for x in y' to get the slope ($m = e$).

Substitute 0 for x in y to get a point on the line $(0, e)$.

$(0, e)$ is the y-intercept (b), and $m = e$; therefore, the line has equation $y = ex + e$.

7. (D) $4 - 2e$

$$v(t) = 2\int e^t\,dt$$
$$v(t) = 2e^t + c$$
$$v(1) = 2e^1 + c = 2$$
$$2e + c = 2$$
$$c = 2 - 2e$$
$$v(t) = 2e^t + (2 - 2e)$$
$$v(0) = 2e^0 + (2 - 2e)$$
$$= 2(1) + (2 - 2e) = 4 - 2e$$

8. (B) $\frac{-3}{7}$

9. (C) -1

10. (D) $\frac{1}{t}$

11. (C) 1

12. (C) $x^3 - \sin(x) + C$

13. (D) $\frac{10}{3}$

14. (D) $\dfrac{\ln\frac{5}{2}}{2}$

$$\int_{-1}^{2} \frac{x}{x^2+1}\,dx = \left[\frac{\ln(x^2+1)}{2}\right]_{-1}^{2}$$
$$= \frac{\ln(2^2+1)}{2} - \frac{\ln(1^2+1)}{2}$$
$$= \frac{\ln\frac{5}{2}}{2} = 0.458$$

15. (B) $\frac{1}{3}(e^5 - e^2)$

16. (A) $5 \sin 10x$

$$y = \frac{1}{2}\Big(1 - \cos 2(5x)\Big) = \frac{1}{2}(1 - \cos 10x)$$

$$\frac{dy}{dx} = (10)\frac{1}{2}(\sin 10x) = 5 \sin 10x$$

17. (D) $x > \frac{1}{6}$

18. (B) $x(5) > x(2)$

19. (E) 100

$$\int_0^5 20\,dt + \int_5^{10}(60 - 8t)\,dt$$

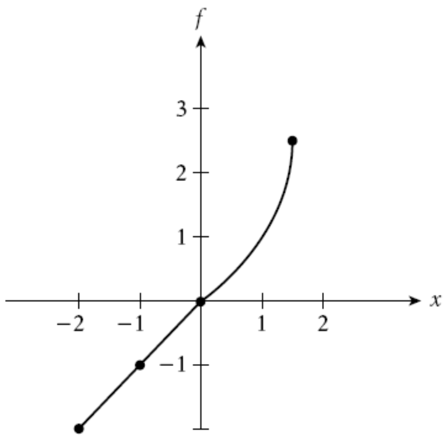
20. (C) 3. Find the equation of a line with $m = 5$ through $(1, 2)$. Then substitute 1.2 into the resulting equation to find that the answer is 3.

21. (B) $y = -12x + 3\pi + 1$

$$\frac{dy}{dx} = \frac{12 \sin 6x}{(1 + \cos 6x)^2}$$

The slope $= -12$, and the point $= \left(\frac{\pi}{4}, 1\right)$, so the line is $y = -12x + 3\pi + 1$.

22. (D)



$f(x) = 1x + C_1$ from -2 to 0.

$f(x) = \frac{x^2}{2} + x + C_2$ from 0 to 2.

23. (A) $-\frac{5}{2}$. By solving $x - 2y = 5$, we can see that the slope of the line is $\frac{1}{2}$. The slope of a line perpendicular would be -2.
Take the derivative of $y = x^2 + 3x + 5$, which is $y' = 2x + 3$.

$2x + 3 = -2$; therefore, $x = -\frac{5}{2}$.

24. (D) I and IV

25. (A) $2xe^{x^2}(x^2 + 1)$
Use the Product Rule to find the derivative.

26. (A) $e^2 - e^1$

$$\int_0^{\frac{\pi}{2}} e^{2-\cos x} \sin x \, dx = e^{2-\cos x}\Big|_0^{\frac{\pi}{2}}$$

27. (B) -2

$$\left(\frac{1}{1 - \frac{1}{2}}\right)\int_{\frac{1}{2}}^{1} -\frac{1}{x^2}\,dx$$

28. (C) 1

Section I
Part B (pages 288–290)

1. (A) $\lim_{x \to 5} f(x)$ exists

2. (C) 2

$f'(y) = 1 - 2 \sin(2x)$
$f''(y) = -4 \cos(2x)$
$-4 \cos(2x) = 0$
$x = -\cancel{\frac{\pi}{4}}, \frac{\pi}{4}, \frac{3\pi}{4}$

3. (B) -1.5

$f(x) = x$ from 0 to 1. Integrate to find $F(x) = \frac{x^2}{2} + C_1$.

It follows that $F(0) = 0 + C_1 = -3$; therefore, $C_1 = -3$, and $F(x) = \frac{x^2}{2} - 3$ from 0 to 1.

$f(x) = 1$ from 1 to 2. Integrate to find $F(x) = x + C_2$.
To be continuous $\frac{1^2}{2} - 3 = 1 + C_2$; therefore, $C_2 = -3.5$.

$F(x) = x - 3.5$ from 1 to 2.

4. (C) I and II

5. (C) -1 and $\frac{5}{3}$

6. (D) 36 cm²/sec

$\frac{dr}{dt} = 6$, $r = 3$, and $A = \pi r^2$

rate of change in area$= \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(6)(3) = 36$ cm²/sec

7. (A) $x = \sqrt{\frac{\pi}{2} - 1} \approx 0.756$

8. (D) 0.568

$$\int_0^{\frac{\pi}{6}} 2 \cdot \cos(x) - 3 \cdot \tan(x)\, dx$$

9. (B) 4π. Use the disc method.

10. (E) There is no relative maximum. Use the First Derivative Test.

11. (B) 0.955. Find the slope between (0, 1) and $\left(\frac{\pi}{3}, 2\right)$, both points on the graph of $f(x) = \sec x$.

12. (D) $\int \frac{5t^2 + 1}{5t} = \frac{1}{5}\ln(|t|) + \frac{1}{2}t^2 + C = v(t)$

$\frac{1}{2}\ln(|1|) + \frac{1}{2}(1^2) + C = 1$

$C = \frac{1}{2}$

Find $v(2)$ by substituting.

13. (C) 10. Graph, connect the points, and divide the graph into trapezoids to find the area.

14. (E) The average rate of change of $f(x)$ on [1, 3] equals $f'(2)$.

It is not (A) because the limit does not exist.

It is not (B) because the limit is 0.

It is not (C) because f is not differentiable at 1.

It is not (D) because $f(1) = 0$.

15. (C) $\int_0^4 \left(\left(4 - \sqrt{x}\right)^2 - 4\right)dx$

$R(x) = 4 - \sqrt{x}$ and $r(x) = 2$. The bound is $0 \le x \le 4$, so the integral is

$$\int_0^4 \left(\left(4 - \sqrt{x}\right)^2 - 4\right)dx.$$

16. (E) 6.770

$$\int_0^2 \sqrt{1 + (3t^2 - 1)^2}\, dt$$

17. (B) $p \ge q$

Section II
Part A (page 291)

1.

(a) $v(t) = \int 20e^{-t/2}dt = -40e^{-t/2} + C$ $v(0) = 0$; therefore, $C = 40$. $v(t) = -40e^{-t/2} + 40$ is constantly increasing. Using $v(10) \approx 39.73$, the starting point for finding the constant for the second integral: $a(t) = -16$, for $t > 10$, and $v(t) = -16t + C$. $C \approx 199.73$, using the value $v(10) \approx 39.73$. $v(t) = -16t + 199.73$ for $t > 10$, is a constantly decreasing function; therefore, the rocket is descending after 10 seconds.	(a) 4 points 2: defining both integrals 1: limits 1: integrands 2: finding $v(t)$ for $\begin{cases} a(t) = 20e^{-t/2}, 0 \le t \le 10 \\ a(t) = -16, \quad t > 10 \end{cases}$ 1: answer
(b) $x(t) = \int -40e^{-t/2} + 40\, dt = 80e^{-t/2} + 40t + C$ Since $x(0) = 0$, $C = -80$. $x(t) = 80e^{-t/2} + 40t - 80$, for $0 \le t \le 10$. $x(10) \approx 320.54$ Now find the other integral and use $x(10)$. $x(t) = \int -16t + 199.73\, dt = -8t^2 + 199.73t + C$ $x(10)$ is about 320.54, so $C = -876.76$.	(b) 3 points 2: defining both integrals 1: limits 1: integrands 1: answer

$x(t) = -8t^2 + 199.73t - 876.76$, for $t > 10$. The second $x(t)$ function continues the ascent of the first, so the maximum will be found using the second $x(t)$ function. To find the maximum value, the derivative of $x(t)$ is $v(t)$, which is zero at ≈ 12.48. $x(12.48) \approx 370$ The rocket will reach a height of 370 feet.	
(c) Use the second $x(t)$ function from part (b) and find where $x(t) = 0$. $-8t^2 + 199.73t - 876.76 = 0$ when $t \approx 5.68$ and 19.28. Because 5.68 is not in the range of the function, $t = 19.28$. $v(19.28) \approx -108.79$, meaning 108.79 ft/sec in the downward direction.	(c) 1 point 1: answer
(d) $x(t) = -8t^2 + 199.73t - 876.76$ for $t > 10$.	(d) 1 point 1: answer

2.

(a) Let V_o = the volume of oil in the can. V_o fills the whole can = $\pi r^2 h = \pi(2)^2 6 = 24\pi$. V_o at the height of 3 in. in the can = $\pi(3)^2 6 = 12\pi$. The volume of oil in the cone = $24\pi - 12\pi = 12\pi$ in^3. Let the depth of the oil in the cone = h_c, radius of the surface of the oil in the cone = r_c, and V_c = the volume of the oil in the cone. $\frac{h_c}{r_c} = \frac{2}{1}$, so $r_c = \frac{1}{2}h_c$. Substitute r_c into the formula for the volume of a cone. $V_c = \frac{\pi\left(\frac{1}{2}h_c\right)^2 h_c}{3} = \frac{\pi h_c^3}{12}$ The height of the oil in the cone when volume of oil in the cone is 12π in.^{3} is $12\pi = \frac{\pi h_c^3}{12}$ $h_c = \sqrt[3]{144}$. Take the first derivative of $V_c = \frac{\pi h_c^3}{12}$ to find rate of change of the depth $\left(\frac{dh}{dt}\right)$. $\frac{dV_c}{dt_c} = \left(\frac{\pi h_c^2}{4}\right)\frac{dh}{dt}$ Substitute known values: $2 = \left(\frac{\pi(\sqrt[3]{144})^2}{4}\right)\frac{dh}{dt}$ $\frac{dh}{dt} = 0.093$ in./sec	(a) 8 points 1: finding the volume of the oil in the cone when oil in the can is 3 inches deep 1: defining radius of surface of the oil in the cone (r_c) in terms of the depth of the oil (h_c) 1: finding the height of the oil in the cone when the oil in the can is 3 inches deep 2: setting up the differential 2: implicit differentiation 1: answer

(b) The volume of the cylinder, given by $\pi r^2 h$, is 24π. (This is also the volume of the oil.)

The volume of a cone is $\frac{1}{3}\pi r^2 h$. Since we are trying to find h, we must relate h to r.

$\frac{r}{h} = \frac{1}{2}$; therefore, $r = \frac{h}{2}$.

The volume of the cone, then, is

$$\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h = \frac{h^3}{12}\pi.$$

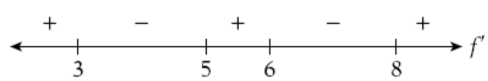
$\frac{h^3}{12}\pi = 24\pi$; therefore, $h \approx 6.6$ in.

(b) 1 point
1: answer

3.

(a) $x = 3$ and $x = 6$

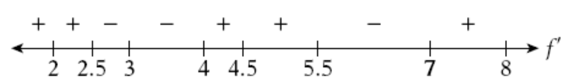
First Derivative Test



Since the f' values change from positive to negative, 3 and 6 are relative maxima.

(a) 2 points
1: answer
1: justification

(b) Second Derivative Test



When $f''(x) > 0$ the graph of f is concave up.

$2 < x < 2.5$
$4 < x < 5.5$
$7 < x < 8$

(b) 3 points
1: $2 < x < 2.5$
1: $4 < x < 5.5$
1: $7 < x < 8$

(c) $x = 2.5, 4, 5.5$

Points of inflection occur when the concavity changes. Use the Second Derivative Test to find the points of inflection. When $f''(x) < 0$ the graph of f is concave down and when $f''(x) > 0$ the graph of f is concave up. Thus, the points of inflection are 2.5, 4, 5.5, and 7.

(c) 2 points
1: answer
1: justification

(d) Yes.

The slopes of the tangent lines between $f(2)$ and $f(3)$ are always positive, indicating an increasing function.

(d) 2 points
1: answer
1: justification

Section II
Part B (page 292)

4.

(a) $\int_0^8 x^{2/3}\,dx = \left.\frac{3x^{2/3}}{5}\right\|_0^8 = \frac{96}{5}$ Half of the volume $= \frac{1}{2} \times \frac{96}{5} = \frac{48}{5}$. $\frac{3k^{5/3}}{5} = \frac{48}{5}$ $k = 4\sqrt[5]{2^2}$ The line is $x = 4\sqrt[5]{2^2}$.	(a) 5 points 2: definite integral 1: limits 1: integrand 3: answer
(b) One third of the volume $= \frac{1}{3} \times \frac{96}{5} = \frac{32}{5}$. $\frac{3p^{5/3}}{5} = \frac{32}{5}$ $p = \frac{8\sqrt[5]{3^2}}{3}$, so the line is $x = \frac{8\sqrt[5]{3^2}}{3}$. Two thirds of the volume $= \frac{32}{5} + \frac{32}{5} = \frac{64}{5}$. $\frac{3q^{\frac{5}{3}}}{5} = \frac{64}{5}$ $q = \frac{8\sqrt[5]{72}}{3}$, so the line is $x = \frac{8\sqrt[5]{72}}{3}$.	(b) 4 points 2: answer $p = \frac{8\sqrt[5]{3^2}}{3}, x = \frac{8\sqrt[5]{3^2}}{3}$ 2: answer $q = \frac{8\sqrt[5]{72}}{3}, x = \frac{8\sqrt[5]{72}}{3}$

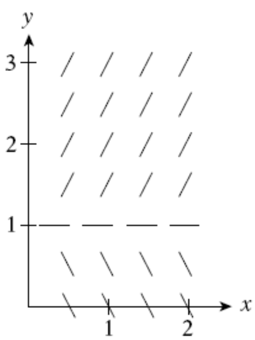
5.

(a) $f(t) = 2t + 2$ $h(x) = \int_1^x 2t + 2\,dt = t^2 + 2t\Big\|_1^x = x^2 + 2x - 3$ $h(2) = 5$ and $h(-1) = -4$	(a) 1 point 1: answer
(b) $h'(x) = \frac{dx}{dt}\left(\int_1^x f(t)dt\right)$, so $h'(x) = f(t)$. If $f(t) < 0$, the function $h(x)$ is decreasing. The interval is [2, 4].	(b) 2 points 1: justification 1: answer
(c) $h'(x) = f(t)$. If $f'(t) = 0$, the function $h(x)$ is at the critical values of h. $x = 2$ and $x = 4$ are the critical values.	(c) 2 points 1: justification 1: answer
(d) Since $h'(x)$ increases from $x = -1$ to $x = 2$, then decreases to $x = 4$, then increases to $x = 5$, there are two inflection points: $x = 3.5$ and $x = 4.5$.	(d) 2 points 1: justification 1: answer

(e) $h'(x) = f(t)$. The slope of $f(t)$ changes from + to − at $x = 2$, so the function $h(x)$ has its absolute maximum at $x = 2$. The slope of $f(t)$ changes from − to + at $x = 4$, so the function $h(x)$ has its absolute minimum at $x = 4$. Thus, the maximum is at $x = 2$ and the minimum is at $x = 4$.	(e) 2 points 1: justification 1: answer

6.

(a)



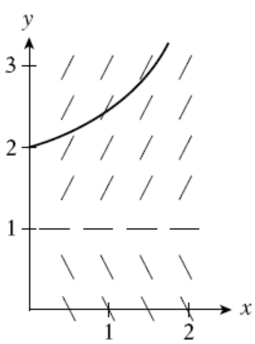
(a) 3 points

1: correct boundaries of slope field

1: zero slope at (1, 1) and (2, 1)

1: positive slope at (1, 2), (2, 2), (1, 3), and (2, 3)

(b)



(b) 3 points

3: answer

(c) $\frac{dy}{dx} = x(y-1)$

$$\int \frac{dy}{y-1} = \int x\,dx$$

$$\ln|y-1| = \frac{x^2}{2} + c$$

$$e^{\ln|y-1|} = e^{\frac{x^2}{2}+c}$$

$$|y-1| = Ae^{\frac{x^2}{2}} \text{ (where } A = e^c\text{)}$$

$$y = Ae^{\frac{x^2}{2}} + 1$$

$$y(0) = Ae^{\frac{0^2}{2}} + 1$$

$$-1 = Ae^0 + 1$$

$$-2 = A$$

$$y = -2e^{\frac{x^2}{2}} + 1$$

(c) 3 points

3: answer