9.7 Taylor and MacLaurin Polynomials

* used to numerically approximate behavior of a function
* MacLaurin is special case of Taylor

MacLaurin:
generated by f at $x=0$

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

Taylor:
generated by f at $x=a$

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \dots$$

① Find 4th degree polynomial that approximates behavior of: $f(x) = \ln(x+1)$ at $x=0$

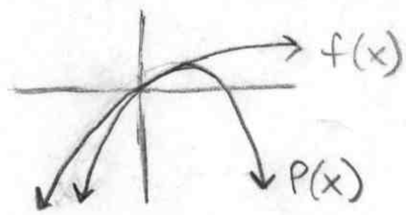
$f(x) = \ln(x+1) \qquad f(0) = 0$

$f'(x) = \frac{1}{x+1} \qquad f'(0) = 1$

$f''(x) = \frac{-1}{(x+1)^2} \qquad f''(0) = -1$

$f'''(x) = \frac{2}{(x+1)^3} \qquad f'''(0) = 2$

$f^4(x) = \frac{-6}{(x+1)^4} \qquad f^4(0) = -6$



$f(.1) = .0953101$

$P_4(.1) = .095308$

* act the same near zero!

$$P_4(x) = 0 + 1x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 \dots$$

$$P_4(x) = x - \frac{x^2}{2} + \frac{1x^3}{3} - \frac{x^4}{4}$$

(2) Find MacLaurin poly for
3 e^{3x} of degree 4.

$f(x) = e^{3x}$ $\quad f(0) = 1$

$f'(x) = 3e^{3x}$ $\quad 3$

$f''(x) = 9e^{3x}$ $\quad 9$

$f'''(x) = 27e^{3x}$ $\quad 27$

$f^4(x) = 81e^{3x}$ $\quad 81$

$$P_4(x) = 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{6} + \frac{81x^4}{24}$$

(3) Find Taylor poly for $f(x) = \frac{2}{x^2}$
2 of degree 4 and center 2.

$f(x) = \frac{2}{x^2} = 2x^{-2}$ $\quad f(2) = \frac{1}{2}$

$f'(x) = \frac{-4}{x^3}$ $\quad -\frac{1}{2}$

$f''(x) = \frac{12}{x^4}$ $\quad \frac{3}{4}$

$f'''(x) = \frac{-48}{x^5}$ $\quad \frac{-48}{32} = \frac{-3}{2}$

$f^4(x) = \frac{240}{x^6}$ $\quad \frac{240}{64} = \frac{15}{4}$

$$P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{4 \cdot 2!}(x-2)^2 - \frac{3}{2 \cdot 3!}(x-2)^3 + \frac{15}{4 \cdot 4!}(x-2)^4$$

$$P_4(x) = \frac{1}{2} - \frac{(x-2)}{2} + \frac{3(x-2)^2}{8} - \frac{1}{4}(x-2)^3 + \frac{5}{32}(x-2)^4$$

More Practice:

8 $f(x) = x^2 e^{-x}$

5 4th degree MacLaurin poly

$f(x) = x^2 e^{-x}$ $\quad f(0) = 0$

$f'(x) = x^2(-e^{-x}) + e^{-x}(2x)$ $\quad 0$

$-x^2 \cdot e^{-x} + 2x \cdot e^{-x}$

$f''(x) =$

$+x^2 \cdot e^{-x} + e^{-x} \cdot (-2x) + 2x(-e^{-x}) + e^{-x}(2)$

$x^2 e^{-x} - 2xe^{-x} - 2xe^{-x} + 2e^{-x}$ $\quad 2$

$f'''(x)$

$-x^2 e^{-x} + 2xe^{-x} - 2(-2xe^{-x} + 2e^{-x}) + -2e^{-x}$

$-x^2 e^{-x} + 2xe^{-x} + 4xe^{-x} - 4e^{-x} - 2e^{-x}$ $\quad -6$

$-x^2 e^{-x} + 6xe^{-x} - 6e^{-x}$

$f^4(x) =$

$x^2 e^{-x} - 2xe^{-x} + 6x(-e^{-x}) + 6e^{-x} + 6e^{-x}$ $\quad 12$

$P_4(x) = 0 + 0x + \frac{2x^2}{2!} + \frac{-6x^3}{3!} + \frac{12x^4}{4!}$

$P_4(x) = x^2 - x^3 + \frac{1}{2}x^4$

(9) $f(x) = \sin x$

4 3rd degree poly, $c = \frac{\pi}{6}$

$f(x) = \sin x \qquad f(\pi/6) = \frac{1}{2}$

$f'(x) = \cos x \qquad \frac{\sqrt{3}}{2}$

$f''(x) = -\sin x \qquad -\frac{1}{2}$

$f'''(x) = -\cos x \qquad -\frac{\sqrt{3}}{2}$

$$P_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{-1}{2 \cdot 2!}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{2 \cdot 3!}\left(x - \frac{\pi}{6}\right)^3$$

$$P_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$$