MEMORIZE! MacLaurin Series:
(Pg. 682)

e^x
$\cos x$
$\sin x$
$\frac{1}{1-x}$

$\boxed{\text{Group}}$

(4) 4^{th} degree MacLaurin Series $c=0$
$f(x) = e^x$

$f(x) = e^x \quad f(0) = 1$
$f'(x) = e^x \quad 1$
$f''(x) = e^x \quad 1$
$f'''(x) = e^x \quad 1$
$f^4(x) = e^x \quad 1$

$P_4(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$
$P_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

(5) $f(x) = \cos x$

$f(x) = \cos x \quad f(0) = 1$
$f'(x) = -\sin x \quad 0$
$f''(x) = -\cos x \quad -1$
$f'''(x) = \sin x \quad 0$
$f^4(x) = \cos x \quad 1$

$P_4(x) = 1 + 0x - \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4$
$P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$

- 3 terms
- 4th degree
- 4th order

(6) $f(x) = \sin x$

$$f(x) = \sin x \qquad f(0) = 0$$
$$f'(x) = \cos x \qquad 1$$
$$f''(x) = -\sin x \qquad 0$$
$$f'''(x) = -\cos x \qquad -1$$
$$f^4(x) = \sin x \qquad 0$$

$$P_4(x) = 0 + 1x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4$$
$$P_4(x) = x - \frac{1}{6}x^3$$

(7) $f(x) = \frac{1}{1-x}$

$$f(x) = (1-x)^{-1} \qquad f(0) = 1$$
$$f'(x) = \frac{+1}{(1-x)^2} = +1(1-x)^{-2} \qquad 1$$
$$f''(x) = -2(1-x)^{-3} \cdot -1 = \frac{2}{(1-x)^3} \qquad 2$$
$$f'''(x) = -6(1-x)^{-4} \cdot -1 = \frac{6}{(1-x)^4} \qquad 6$$
$$f^4(x) = -24(1-x)^{-5} \cdot -1 = \frac{24}{(1-x)^5} \qquad 24$$

$$P_4(x) = 1 + x + \frac{2}{2!}x^2 + \frac{6}{3!}x^3 + \frac{24}{4!}x^4$$
$$P_4(x) = 1 + x + x^2 + x^3 + x^4$$

9.7 – 9.8 Building a Series

* MEMORIZE:

1) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ for all x

2) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$ for all x

3) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$ for all x

4) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$ for $-1 < x < 1$

Using Known MacLaurin Series, construct first 3 nonzero terms and general term:

① $f(x) = \sin 2x$

* * $\sin x$ $\quad x - \frac{x^3}{3!} + \frac{x^5}{5!} \quad \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

* $\sin 2x$ $\quad 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \quad \sum \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!}$

② $f(x) = \sin x^3$

$$x^3 - \frac{(x^3)^3}{3!} + \frac{(x^3)^5}{5!}$$

$$x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} \quad \sum \frac{(-1)^n (x^3)^{2n+1}}{(2n+1)!}$$

③ $f(x) = e^{2x}$

* * e^x $\quad 1 + x + \frac{x^2}{2!} \quad \sum \frac{x^n}{n!}$

* e^{2x} $\quad 1 + 2x + \frac{(2x)^2}{2!} \quad \sum \frac{(2x)^n}{n!}$

$1 + 2x + \frac{4x^2}{2!}$

④ $f(x) = 7x \cdot e^x$

* $\quad 7x + 7x^2 + \frac{(7x)^3}{2!} \quad \sum \frac{(7x) \cdot x^n}{n!}$

$= \sum \frac{7x^{n+1}}{n!}$

⑤ $f(x) = x^2 \cos x$

* * $\cos x$ $\quad 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \sum \frac{(-1)^n x^{2n}}{(2n)!}$

* $x^2 \cos x$

$x^2 - \frac{x^4}{2!} + \frac{x^6}{4!} - \frac{x^8}{6!} \quad \sum \frac{(-1)^n x^2 \cdot x^{2n}}{(2n)!}$

$\sum \frac{(-1)^n x^{2n+2}}{(2n)!}$

(6) $f(x) = \dfrac{x}{1-x^2}$

* $\dfrac{1}{1-x} \quad 1 + x + x^2 + x^3 + \ldots \quad x^n$

* $\dfrac{1}{1-(x^2)} = 1 + x^2 + x^4 + x^6 + \ldots \quad x^{2n}$

* $\dfrac{x}{1-(x^2)} = x + x^3 + x^5 + x^7 + \ldots \; x \cdot x^{2n}$
$= x^{2n+1}$

(7) $f(x) = e^{-2x}$

* $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots \quad \dfrac{x^n}{n!}$

* $e^{-2x} = 1 - 2x + \dfrac{4x^2}{2!} - \dfrac{8x^3}{3!} + \ldots \quad \dfrac{(-2x)^n}{n!}$

(8) $f(x) = \dfrac{1}{1+x}$

*

* $\dfrac{1}{1-x} \quad 1 + x + x^2 + x^3 + \ldots \quad x^n$

* $\dfrac{1}{1+x} = \dfrac{1}{1-(-x)} = 1 - x + x^2 - x^3 + x^4$
$-x^5 \ldots \quad (-x)^n$

(9) $f(x) = \cos(x^2)$

* $\cos x \quad 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \frac{(-1)^n x^{2n}}{(2n)!}$

* $\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \ldots \frac{(-1)^n (x^2)^{2n}}{(2n)!}$

$\frac{(-1)^n (x^{4n})}{(2n)!}$

(10) $f(x) = x\sin(3x)$

* $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

* $\sin(3x) = 3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \frac{(3x)^7}{7!} + \ldots \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}$

* $x\sin(3x) = 3x^2 - \frac{27x^4}{3!} + \frac{243x^6}{5!} + \ldots$

$\frac{(-1)^n x (3x)^{2n+1}}{(2n+1)!}$