9.5 Alternating Series Test

*AST:

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n \cdot a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n+1} \cdot a_n$$

converge if following 2 conditions are met

1. $\lim_{n\to\infty} a_n = 0$ 2. $a_{n+1} \le a_n$, for all n

1) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{10^n}$ $\lim_{n\to\infty} \frac{n}{10^n} = 0$ converges

2) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$ $\lim_{n\to\infty} \frac{n^2}{n^3+4} = 0$ converges

3) $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$ $\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2}$ diverges

4) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^{3/4}}$ $\lim_{n\to\infty} \frac{1}{n^{3/4}} = 0$ converges

5) $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} \cdots$

$\sum_{n=7}^{\infty} \frac{4}{n} (-1)^{n+1}$ $\lim_{n\to\infty} \frac{4}{n} = 0$ converges

* Absolute and Conditional Convergence:

- If $\sum |a_n|$ converges, then $\sum a_n$ converges

- Def: Absolute and Conditional Convergence

1. $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges
2. $\sum a_n$ is conditionally convergent if $\sum a_n$ converges but $\sum |a_n|$ diverges

Absolutely converges, Conditionally converges, or Diverges?

1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$ $\quad \left|\frac{(-1)^{n+1}}{n^{3/2}}\right| = \frac{1}{n^{3/2}}$ absolutely converges

~~2)~~ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{(n+10)}$ $\quad \left|\frac{(-1)^{n+1}(2n+3)}{(n+10)}\right| = \frac{2n+3}{n+10}$

$\displaystyle\lim_{n\to\infty} \frac{2n+3}{n+10} \overset{L'H:}{=} 2 \neq 0$ divergent

3) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ $\quad \left|\frac{(-1)^{n+1}}{n^{1/4}}\right| = \frac{1}{n^{1/4}}$ diverges

AST: $\displaystyle\lim_{n\to\infty} \frac{1}{n^{1/4}} = 0$ converges $\therefore$ conditionally convergent

4) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n}}{\sqrt{n}}$ $\quad \left|\frac{(-1)^{n}}{\sqrt{n}}\right| = \frac{1}{n^{1/2}}$ diverges

AST: $\displaystyle\lim_{n\to\infty} \frac{1}{n^{1/2}} = 0$ converges $\therefore$ conditionally convergent