What do you notice about the limits and the vertical asymptotes of your functions? Write a rule that describes the relationship. If $f(x) \to \pm\infty$ as $x \to c$ from Right or Left, then $x = c$ is a VA.

6) Recall the 3 cases that produce horizontal asymptotes: BOSTON

Case	Horizontal Asymptote
Degree of Denominator > Degree of Numerator	$y = 0$
Degree of Numerator = Degree of Denominator	$y = \frac{\text{lead coeff}}{\text{lead coeff}}$
Degree of Numerator > Degree of Denominator	none

For each function, determine the horizontal asymptote.

7) $f(x) = \frac{5x^2-2}{x^2}$ $y = 5$

8) $f(x) = \frac{2x-1}{x+1}$ $y = 2$

9) $f(x) = \frac{4x^2-1}{3x^2+5}$ $y = \frac{4}{3}$

10) $f(x) = \frac{4x-1}{3x^2+5}$ $y = 0$

11) $f(x) = \frac{4x^3-1}{3x^2+5}$ none

Whole Group: Now let's evaluate the limits of the functions above.

7) $\lim_{x\to\infty} f(x) =$ $\frac{5x^2-2}{x^2} = 5$

8) $\lim_{x\to\infty} f(x) =$ $\frac{2x-1}{x+1} = 2$

9) $\lim_{x\to-\infty} f(x) =$ $\frac{4x^2-1}{3x^2+5} = \frac{4}{3}$

10) $\lim_{x\to\infty} f(x) =$ $\frac{4x-1}{3x^2+5} = \frac{4}{3x}$ → 0 ↑ small; ↳ big

11) $\lim_{x\to-\infty} f(x) =$ $\frac{4x^3-1}{3x^2+5} = \frac{4x}{3} =$ big neg $= -\infty$

12) What do you notice about the horizontal asymptotes and the limits of your functions as $x \to \pm\infty$. Write a rule that describes the relationship.

If $\lim_{x\to-\infty} f(x) = L$ or $\lim_{x\to\infty} f(x) = L$, then $y = L$ is a HA